

## The YREC Stellar Evolution Code: Public Data Release

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### ABSTRACT

In this paper we present the public release of the Yale Rotating Evolution Code (YREC). YREC is a stellar evolution code that covers brown dwarfs and stars across a wide range of masses, and evolutionary states from the pre-MS through helium burning. We summarize the key ingredients of the code, document the code performance, and discuss its strengths and limitations. We present libraries of input files, documentation, sample use cases, and scripts. In addition to usage as a research tool, we highlight the utility of the code for educational purposes.

*Keywords:* Stellar astronomy (1583) — Solar physics (1476)

### 1. INTRODUCTION

The theory of stellar structure and evolution is deep and beautiful. Its bedrock lies in fundamental physical principles. The equations of stellar structure are expressions of the conservation of mass, energy, and momentum, complemented by the second law of thermodynamics. The remarkably short free-fall timescale ensures that hydrostatic balance is a superb approximation for almost all stars in almost all circumstances, greatly simplifying the general problem. The transport and generation of energy are also closely matched in stars, so stars are near-equilibrium structures. As a consequence, stellar structure, or the solution of the equations at an instant in time, is a tractable problem. It is only over oceans of time that the graceful arc of stellar evolution proceeds, driven by the slow but irreversible transformation of light species into heavy ones by nuclear reactions.

What might seem at first to be an impossible problem can then be cast in a straightforward form. It simply requires tools from the full arsenal of modern physics—quantum mechanics, thermodynamics, statistical mechanics, electricity and magnetism, nuclear physics, and fluid mechanics. The transport of radiation through matter can be captured by an effective mean free path of a photon, expressed as an opacity. Nuclear reaction rates and the equation of state can similarly be defined as functions of the local temperature, density and composition. Even difficult physical problems, such as the transport of energy by turbulent convective motions, can be collapsed into a local prescription. Solving these equations, of course, is not the same as setting them up.

Analytic work has a distinguished history in stellar theory, but the limitations of these tools became very clear at an early stage. The microphysics are clearly posed questions, but not ones with answers in convenient analytical forms. The correct solution to the equations need not be easy to compute. The dawn of the computer age in the 1950s and 1960s led directly to the first

62 generation of numerical stellar structure codes. Astro-  
 63 physicists were able to use compact codes and kilobytes  
 64 of memory, deployed with punch cards, to decode the  
 65 life cycle of stars. A number of independent codes, still  
 66 in use today, were developed. In this paper we present  
 67 the public release of the Yale Rotating Stellar Evolution  
 68 Code (YREC), which had its origin in this era.

69 Despite the variety of stellar codes that were devel-  
 70 oped, only a few major codes are public: the MESA  
 71 code (B. Paxton et al. 2011, 2013, 2015, 2018, 2019; A. S.  
 72 Jermyn et al. 2023), CESAM (P. Morel 1997; P. Morel &  
 73 Y. Lebreton 2008; L. Manchon et al. 2025), the Eggleton  
 74 STARS code (P. P. Eggleton 1972; P. P. Eggleton et al.  
 75 2011), and (recently) the ESTER code (R. White et al.  
 76 2025). MESA has been particularly high in impact and  
 77 is widely used. This is not because other authors were  
 78 unwilling to share and compare results: a good example  
 79 of detailed code comparisons within the community can  
 80 be found in J. N. Bahcall & M. H. Pinsonneault (1992)  
 81 for solar neutrinos. Instead, the scarcity of public tools  
 82 reflects the mismatch between the era in which those  
 83 codes were developed and the time frame when the open  
 84 source movement blossomed. Proprietary codes are also  
 85 typically not built with the needs of a large and diverse  
 86 user base in mind. Some are carefully curated gems.  
 87 But walled gardens can also be filled with weeds. Re-  
 88 leasing such codes therefore requires careful boundaries,  
 89 which we lay out below.

90 Users are free to modify the YREC source code, but  
 91 the code as written was not designed with modular user  
 92 inputs in mind. In future releases, we will prioritize  
 93 making the code easier to alter. YREC will also do what  
 94 you tell it to do, without second-guessing. This means  
 95 that the code can and will crash if you assign, for exam-  
 96 ple, a super-critical rotation rate. The documentation  
 97 does discuss known failure modes and how to diagnose  
 98 them. Many of the inputs for YREC were designed for  
 99 specific science cases, and the code is not guaranteed to  
 100 perform well outside of that domain, where the assump-  
 101 tions break down. Finally, YREC is written in glorious  
 102 Fortran, F77 to be specific, but it is F90 compatible.  
 103 With these cautions in mind, the modern open source  
 104 movement has proven to be fruitful and powerful, and  
 105 we believe that YREC will be a useful tool for research  
 106 and education.

### 107 1.1. *Why YREC?*

108 YREC is a stellar evolution code optimized for the  
 109 study of hydrostatic stars from the pre-main sequence  
 110 through to the helium burning phase of evolution.  
 111 YREC can also be used to study giant planets and white  
 112 dwarfs. It is fast, precise, and accurate in the domains

113 where it functions. YREC has been extensively used for  
 114 both stellar population and stellar physics studies, in-  
 115 cluding high-precision solar models (C. Basinger et al.  
 116 2024).

117 YREC uses modern input microphysics (equation  
 118 of state, energy transport, energy generation, surface  
 119 boundary conditions) with a variety of user-configurable  
 120 options. YREC is a relaxation code, and as written,  
 121 it requires a starting model to run. These models can  
 122 then be relaxed to a different set of initial conditions. It  
 123 has the option to include important physical processes  
 124 not considered in classical stellar models. For example,  
 125 YREC includes the structural effects of rotation using  
 126 a pseudo-2D approximation. The treatment of angular  
 127 momentum evolution is flexible, including star-disk cou-  
 128 pling, rotationally induced mixing, internal angular mo-  
 129 mentum transport, angular momentum loss from mag-  
 130 netized winds, and the ability to model different rotation  
 131 profiles in convective regions. It also includes gravita-  
 132 tional settling and the structural effects of starspots and  
 133 their associated magnetic fields.

### 134 1.2. *Outline of the Paper*

135 We describe the input physics and numerical approach  
 136 of the YREC code in Section 2. We also note similarities  
 137 and differences with the public MESA code.

138 Stellar evolution codes adopt a set of input physics  
 139 and run a single initial condition (mass, composition,  
 140 rotation rate, and age) to a prescribed end state. A test  
 141 suite of runs highlights the various capabilities of the  
 142 code. The test suite is also an essential tool for code de-  
 143 velopment, as it allows users to check that code changes  
 144 have not caused unanticipated problems. In Section 3,  
 145 we present our test suite, sample applications, and com-  
 146 parisons with other codes in the literature. Scripts and  
 147 helper codes are discussed in Section 4. In the discus-  
 148 sion and summary, we lay out the current domains where  
 149 the code is recommended for usage and future plans for  
 150 upgrades. Details of how to access the code repository,  
 151 input files, documentation, and other resources are given  
 152 in the appendices.

## 153 2. PHYSICS AND NUMERICS OF THE YREC 154 CODE

155 YREC has been in use as a stellar structure and evo-  
 156 lution code for more than 60 years. The code was first  
 157 described in R. B. Larson & P. R. Demarque (1964),  
 158 and subsequently updated and expanded as described  
 159 by M. J. Prather (1976). The most recent summaries  
 160 of the standard model physics are P. Demarque et al.  
 161 (2008), which included the addition of output for com-  
 162 puting pulsations; and J. L. van Saders & M. H. Pin-  
 163 sonneault (2012), which includes a series of updates to

the input physics. Microscopic diffusion was added by J. N. Bahcall & M. H. Pinsonneault (1992); J. N. Bahcall et al. (1995), along with a complete overhaul of the treatment of nuclear reaction rates. We also note that the DSEP code, used for a popular set of isochrones (A. Dotter et al. 2008), was based on YREC and shares much of its structure.

M. H. Pinsonneault et al. (1989) added rotation and rotationally induced mixing. There were then a series of updates to the angular momentum evolution model. This included updates to the angular momentum loss model (A. Krishnamurthi et al. 1997; J. L. van Saders & M. H. Pinsonneault 2013), updates to the rotationally induced mixing model (M. H. Pinsonneault et al. 2002; G. Somers & M. H. Pinsonneault 2016), and the inclusion of the structural effects of starspots (G. Somers & M. H. Pinsonneault 2015; G. Somers et al. 2020).

### 2.1. Overall Code Structure and Repositories

YREC is a relaxation code that can be run in 2 modes. An evolving run takes an input model and runs to a specified end state. A rescaling model uses a prior solution that is close to the desired conditions and perturbs it. Rescaling can be done in the fully convective pre-main sequence (pre-MS), the zero-age main sequence (ZAMS), or the zero-age core He burning state (ZAHB). For the pre-MS, there is no time independent solution of the structure equations. A pre-MS model must therefore evolve in time when rescaling. On the ZAMS and the ZAHB, the code rescales with a zero time step. We note that many useful stop conditions are not enabled for rescaling runs of any form. If rescaling is desired, it is therefore recommended to do runs in two stages: an initial rescaling run followed by an evolving run with a stop condition.

Running YREC requires the user to download the source code, and then to use a fortran compiler to compile the subroutines and link them. The code, documentation, and instructions on how to run the code can be found at <https://github.com/yreclab/yrec>.<sup>11</sup> YREC also requires libraries of input tables and starting models, which are available on the repository under the `input` and `startmodels` directories, respectively. These data are also available in the YREC Zenodo repository (<https://zenodo.org/communities/yrecmodels>).

The code uses two namelists that include global run instructions, file input and output paths, numerical controls, and an extensive list of user-controllable options for the input physics. Running the code also requires a

converged starting model and tabulated input data for the microphysics. Suites of models can be run using scripts, and the output can be adopted to investigate a range of astrophysical problems.

The github repository contains a `testsuite` directory that includes the namelists and output for the test suite, which can also be found on the Zenodo repository. The Zenodo repository also has sections for scripts, classroom exercises, a base model grid covering a range of masses and metallicities, and sample cases used to generate all of the figures and tables in this paper.

### 2.2. Numerical Methods

YREC solves the equations of stellar structure using the Henyey technique. In this method, the four coupled equations of stellar structure are converted into finite difference equations. A valid solution of the structure equations at a given time is then perturbed to account for composition changes and gravitational potential energy release. The start-of-timestep structure determines the locations of convective regions. It is also used to compute nuclear reaction rates, which are then used to infer composition changes in an implicit scheme. Composition changes drive changes in the structure variables, which are inferred, along with their derivatives, in microphysics routines. As an example, the conversion of hydrogen to helium lowers the number of free particles per gram, reducing pressure at fixed temperature and density. The model will therefore require compensating changes in temperature and density to restore hydrostatic equilibrium, which in turn cause feedback for other state variables. The code then solves iteratively for the required changes in the structure variables until it reaches a user-specified set of numerical precision criteria.

Stellar evolution is a two point boundary value problem. At the center, YREC uses a Taylor series expansion to map the inner shell to the central values. This approach is widely used in stellar codes, as it avoids the divergence of logarithmic structure variables at zero mass and radius. Close to the surface the assumption of a short photon mean free path becomes invalid. All stellar evolution codes therefore impose a solution of the radiative transfer equations as an atmospheric boundary condition. This can either be a direct integration, such as a gray atmosphere, or a table look-up from a grid of full model atmosphere calculations.

An important difference between YREC and MESA is the treatment of the outer boundary condition below the atmosphere. For a two point boundary value problem, the governing equations must be integrated both inwards from the surface and outwards from the center.

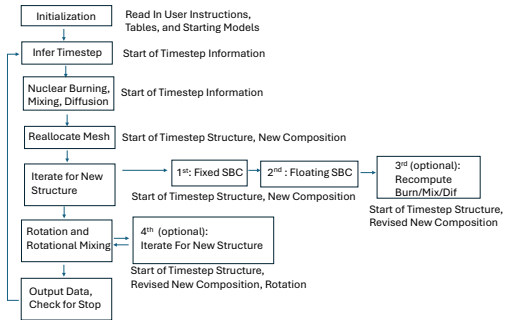
<sup>11</sup> The release paper version of the code can be found in the [https://github.com/yreclab/yrec/tree/yrec\\_2026\\_release](https://github.com/yreclab/yrec/tree/yrec_2026_release) branch.

263 The two solutions and their derivatives are matched at  
 264 a fitting point, and the location of the fitting point is a  
 265 numerical choice that reflects tradeoffs. The base of the  
 266 atmosphere is a possible location for the fitting point,  
 267 but this is not required. A deeper fitting point is usually  
 268 more stable, but the inwards integration usually makes  
 269 simplifying assumptions. For example, in YREC the lu-  
 270 minosity is constant in the envelope integration. MESA  
 271 models typically adopt a fitting point at the base of the  
 272 atmosphere.<sup>12</sup>

273 YREC adopts an outer fitting point well below the  
 274 photosphere. At this fitting point, the structure vari-  
 275 ables  $P$ ,  $T$ ,  $R$  and  $L$  from the interior solution and  
 276 their derivatives must be matched to the correspond-  
 277 ing values from a downwards envelope integration. The  
 278 code also needs information on how these envelope in-  
 279 tegration properties change as a function of  $L$  and  $T_{\text{eff}}$ .  
 280 YREC therefore integrates three solutions close to the  
 281 model  $L$  and  $T_{\text{eff}}$ , and interpolates between them in this  
 282 envelope triangle as the model solution evolves. If the  
 283 solution moves outside of the triangle the code will flip  
 284 it to re-center the triangle around the current model lo-  
 285 cation.

286 Although very little mass is usually in this envelope,  
 287 typically one part in  $10^4$  or less, this is a regime where  
 288 the pressure and density are dropping very rapidly in  
 289 stars. As a result, not retaining the outer layer structure  
 290 yields a significant increase in computing speed. The  
 291 structure equations are also more stable with a deeper  
 292 fitting point. The cost is that information on the de-  
 293 tailed outer layer physics is not stored in the interiors  
 294 model. YREC therefore has options to post-process an  
 295 envelope solution, for both informational purposes and  
 296 for practical applications such as computing oscillation  
 297 frequencies.

298 YREC solves the structure equations in up to four it-  
 299 erative loops, two of which are mandatory. In the first  
 300 loop, the  $L$  and  $T_{\text{eff}}$  at which the surface boundary con-  
 301 ditions (SBCs) are evaluated is held fixed. In the second,  
 302 main loop, the location of the “envelope triangle” is al-  
 303 lowed to change dynamically. The third, optional, level  
 304 of iteration allows the code to use the end-of-timestep  
 305 structure to compute the composition change, rather  
 306 than the start-of-timestep structure. Microscopic dif-  
 307 fusion and mixing from rotation are computed after the  
 308 model structure has converged in a separate step, so the  
 309 converged structure in the first through third level of  
 310 iteration does not account for all of the changes in the



**Figure 1.** A schematic illustration of the steps in the YREC code.

311 composition profile or for changes in the structural ef-  
 312 fects of rotation during a timestep. The fourth level of  
 313 iteration allows the code to iterate between the rotation,  
 314 diffusion, and structure solutions. The overall program  
 315 flow is illustrated in Figure 1.

316 For each model, the code begins by reallocating the  
 317 spatial mesh points. Mesh points are allocated based on  
 318 the requirement that the state variables not change from  
 319 one point to the next by more than a critical value. In  
 320 the deep core, mass is usually the limiting factor. Lumi-  
 321 nosity governs the spacings in nuclear burning regions,  
 322 while pressure is the governing ingredient for envelopes.  
 323 Points with composition or rotation rate discontinuities  
 324 can be flagged to prevent rezoning across them.

325 At the end of the timestep, the code computes a series  
 326 of limits for the next timestep; the most stringent rele-  
 327 vant criterion is used. The code uses the instantaneous  
 328 rate of nuclear reactions to set a limit on the timestep  
 329 for the next model. It also computes the rate of change  
 330 of the structure variables, from start to end of timestep,  
 331 to set another limit on the timestep; this is adopted if  
 332 shorter than the nuclear burning timestep. This second  
 333 criterion is essential for pre-MS models, for which the  
 334 energy source is gravitational potential energy. For the  
 335 main sequence and beyond, this alternative timestep cri-  
 336 terion is disabled; otherwise it induces very small time  
 337 steps in the shell hydrogen burning stage.

338 The code then writes data to output files as requested,  
 339 and checks whether a stop condition has been achieved.  
 340 If the run continues, the code then returns to the start  
 341 of the loop described above.

### 342 2.3. Classical Stellar Model Physics

343 Stellar evolution codes are designed for exploration.  
 344 It is therefore natural that YREC has many options for  
 345 how to treat important physical processes. The YREC  
 346 code has evolved over time, and it has retained back-  
 347 wards compatibility with historical choices for the input  
 348 physics. In this paper we do not dwell on these charm-

<sup>12</sup> For some tabulated model atmospheres, there is a choice in MESA to adopt a deeper fitting point, up to  $\tau = 100$ , but this is still relatively close to the surface.

ing relics, turning our attention instead to the options recommended for use today.

### 2.3.1. *The Equation of State*

The equation of state relates pressure to temperature, density, and composition, and it also encodes extensive thermodynamic information. As such, it is used for a wide range of purposes in stellar evolution codes. The current version of YREC has three different choices for an equation of state.

The recommended equation of state is the 2005 update of the OPAL EoS (F. J. Rogers & A. Nayfonov 2002). We note that this EoS has a fixed heavy element content. Users can generate and use input tables with different metal contents, but the metallicity is fixed for a given run of the code. In the high-density and cool domain, the OPAL table does not cover some of the phase space of interest. The D. Saumon et al. (1995), or SCV, equation of state is a good alternative here.

The default equation of state, which we refer to as the Yale EoS (M. J. Prather 1976), is available across the full phase space. It includes radiation pressure and degeneracy pressure. The latter is solved for using tabulated solutions to the Fermi integrals, and it includes relativistic corrections. For the gas pressure, the code solves the Saha equations for hydrogen, helium, and metals at low temperature. Above a user-specified threshold, it assumes a fully ionized plasma. The Debye-Huckle correction is also included, to account for electrostatic effects. This EoS is extremely fast, with smooth analytic derivatives, and we recommend its usage for educational applications. However, the code assumptions break down for cool dense plasmas, especially for lower main sequence stars. Most published applications of YREC therefore use tabulated EoS data that includes a more sophisticated treatment, particularly for collective plasma effects.

YREC has a tie-breaker algorithm. It will use the OPAL EoS when possible if it is selected as an option. Outside the table domain, it then checks if the SCV EoS is available (if specified); the Yale EoS is the backup if neither is available. In all cases the code ramps between solutions close to table edges. We caution that thermodynamic consistency is not in general preserved in these cases, which can cause numerical issues in model domains close to table edges. The implementation is different in detail than that in MESA, but both codes use a broadly similar approach.

### 2.3.2. *Radiative and Conductive Energy Transport*

The opacity of matter to light depends, in complicated ways, on the composition, density, and temperature.

The calculation of opacities requires several steps. It is necessary i) to determine the ionic state of the absorbing medium as well as the population of the atomic levels of its constituents, ii) to calculate the absorbing coefficients for different processes (photo-excitation, photoionization), and iii) to include plasma effects (the ions cannot be considered as isolated). Accuracy versus completeness is still at play nowadays as it was 30 years ago. The atomic systems considered today may be much larger, thanks to the computer power available, but the calculations still require approximations and compromises since they cannot be infinite.

In stellar interiors, local thermal equilibrium (LTE) is a very good approximation since the mean free path of a photons is very small. In this case, we can use a diffusion approximation for energy transport, with a diffusion coefficient that is the product of  $c/3$  and an effective photon mean free path. The latter can be well approximated by a weighted average of the monochromatic opacity of the mixture of elements present in the stellar plasma. This Rosseland mean opacity is defined, per gram, by

$$\frac{1}{\kappa} = \int \frac{1}{\kappa_\nu} \frac{\partial B_\nu}{\partial T} d\nu \quad (1)$$

with  $B_\nu$  being the Planck function characterizing the energy distribution of blackbody radiation and  $\kappa_\nu$  being the monochromatic opacity of the mixture.

$$\kappa_\nu = \sum_{s=species} f_\nu(s) (\kappa_{BB}^s(s) + \kappa_{BF}^s(s)) + \kappa_{FF} \quad (2)$$

with  $f_s$  being the fractions of species  $s$ , and where BB stands for bound-bound transitions (photo excitation processes), BF stands for bound-free transitions (photo ionization processes) and FF stands for free-free transitions (inverse Bremsstrahlung processes). For the atomic opacities, the main providers are the OPAL project (C. A. Iglesias & F. J. Rogers 1996) and the Opacity Project (N. R. Badnell et al. 2005), hereafter OP. For these, each of the processes (BB and BF) have to be calculated for each ion of each species and combined with a detailed equation of state. If we consider the opacities the OP distributes over its website (<https://cdsweb.u-strasbg.fr/topbase/testop/home.html>) this corresponds to 241 ions for the 17 species (H to Ni). Since the Rosseland mean opacity is a harmonic mean, the details of the lines and resonances as well as the plasma effects (mainly their broadening) are crucial and can lead to large differences depending on the resolution of the photo-ionization cross sections, line transitions, and the broadening theory.

Molecular opacities require a molecular EoS, which is far more complex than the atomic case. The same holds

448 true for the molecular energy levels and transitions rela-  
 449 tive to the atomic ones. However, for astrophysical condi-  
 450 tions molecules are rare above temperatures of order  
 451 6000 K, and hydrogen ionization is the dominant opac-  
 452 ity source in any case at temperatures of order  $10^4$  K.  
 453 YREC, like other stellar evolution codes, therefore uses  
 454 dedicated molecular opacity tables computed only for  
 455 low temperatures.

456 YREC uses three families of opacity tables: conduc-  
 457 tive, atomic, and molecular. Our conductive opacities  
 458 are taken from *S. Cassisi et al. (2007)*. We use molec-  
 459 ular opacities from *J. W. Ferguson et al. (2005)* below  
 460  $\log T = 4.0$ , atomic opacities above  $\log T = 4.1$ , and  
 461 a ramp between the two for intermediate temperatures.  
 462 There are three families of atomic opacities that the code  
 463 can use—OPAL tables for carbon and oxygen-rich mix-  
 464 tures (*C. A. Iglesias & F. J. Rogers 1993*), OPAL tables  
 465 for H, He, and metal mixtures (*C. A. Iglesias & F. J.*  
 466 *Rogers 1996*) and OP tables for H, He, and metal mix-  
 467 tures (*N. R. Badnell et al. 2005*). The first set of opac-  
 468 ities are used in helium-burning regions, where the heavy  
 469 element mixture changes significantly. For both OP and  
 470 OPAL, the heavy element mixture is held fixed, and the  
 471 opacity is interpolated as a function of  $X$ ,  $Z$ ,  $\rho$  and  $T$ .  
 472 These tables, with these assumptions, are widely used  
 473 in stellar models, and there are web tools available to  
 474 build tables for user-specified heavy element mixtures  
 475 (for the Opacity Project, [https://opacity-cs.obspm.fr/](https://opacity-cs.obspm.fr/opacity/index.html)  
 476 [opacity/index.html](https://opacity-cs.obspm.fr/opacity/index.html)). YREC uses tables constructed for  
 477 a variety of heavy element mixtures, typically tied to  
 478 different assumed solar abundance measurements. Our  
 479 base heavy element mixture for this paper is *N. Grevesse*  
 480 *& A. J. Sauval (1998)*, which produces solar models in  
 481 good agreement with helioseismology. We also include  
 482 tables based on the published solar mixtures of *E. Magg*  
 483 *et al. (2022)*, *M. Asplund et al. (2021)*, and *K. Lodders*  
 484 *et al. (2025)*.

### 485 2.3.3. Convection and Convective Boundary Layer Physics

486 YREC uses the mixing length theory (MLT) of con-  
 487 vection (*E. Böhm-Vitense 1958*) to infer the actual tem-  
 488 perature gradient in regimes that are unstable against  
 489 convection. This requires knowledge of the radiative and  
 490 adiabatic temperature gradients. The code considers the  
 491 Schwarzschild criterion for instability, not the Ledoux  
 492 criterion. See *P. Demarque et al. (2008)* for a fuller dis-  
 493 cussion of this point.

494 The effective convective flux can be modified in models  
 495 that include starspots (section 2.5 below). In convective  
 496 cores the specific heat is quite high and there is an op-  
 497 tion to skip MLT calculations; in this case the adopted  
 498 temperature gradient is the adiabatic one.

499 YREC adopts a step overshoot model, expressed as a  
 500 number of pressure scale heights at the boundary. Over-  
 501 shoot can be enabled separately for cores, envelopes, and  
 502 intermediate regimes.

503 The code has a second, recommended control, which  
 504 limits the overshoot domain to a fraction of the physical  
 505 size of the core (*J.-H. Woo & P. Demarque 2001*). If  
 506 adopted, the code selects the smaller of these two mea-  
 507 sures. Checking against the size of the core is important  
 508 to properly model small convective cores, because the  
 509 pressure scale height formally diverges at zero radius  
 510 and becomes very large for small cores.

511 Overshoot as treated here is more properly overmix-  
 512 ing, as the radiative temperature gradient is retained.  
 513 There is a code option to enforce adiabatic overshoot.

514 YREC also considers semi-convection, using the *V.*  
 515 *Castellani et al. (1971)* approach. We are in the process  
 516 of updating this to a newer approach, but do not yet  
 517 have a publicly supported alternative.

### 518 2.3.4. Nuclear Reaction Rates

519 Nuclear reaction rates are important for both energy  
 520 generation and composition change. YREC is designed  
 521 for the hydrogen and helium burning stages of evolution.  
 522 It also includes light element burning ( ${}^6\text{Li}$ ,  ${}^7\text{Li}$ ,  ${}^9\text{Be}$ ) in  
 523 the trace element approximation, where the contribution  
 524 to energy generation is neglected. This is an excellent  
 525 approximation for these rare species. The code includes  
 526 full non-equilibrium burning for  ${}^1\text{H}$ ,  ${}^3\text{He}$ ,  ${}^4\text{He}$ ,  ${}^{12}\text{C}$ ,  ${}^{13}\text{C}$ ,  
 527  ${}^{14}\text{N}$ ,  ${}^{16}\text{O}$ , and  ${}^{18}\text{O}$ .

528 Deuterium burning is not explicitly traced in the pp  
 529 chain; however, for young stars non-equilibrium burning  
 530 of  ${}^2\text{H}$  into  ${}^3\text{He}$  is important. The code therefore does  
 531 track the burning, and associated energy generation, for  
 532 pre-MS deuterium. It does not, however, treat the pp  
 533 reaction as a source, so deuterium in the models simply  
 534 goes to zero, typically early in the pre-MS phase.

535 For dense nuclear reaction grids, most stellar evolu-  
 536 tion codes use either analytic fits to reaction rates or a  
 537 table look-up. The NACRE (*C. Angulo et al. 1999*) and  
 538 NACRE-2 (*Y. Xu et al. 2013*) compilations of rates are  
 539 commonly used. By default, MESA nuclear reaction  
 540 rates are from JINA REACLIB (*R. H. Cyburt et al.*  
 541 *2010*) and NACRE (*C. Angulo et al. 1999*). The YREC  
 542 code uses this approach for He burning, adopting the  
 543 energy yields and rates of the *G. R. Caughlan & W. A.*  
 544 *Fowler (1988)* compilation for triple  $\alpha$ ,  ${}^{12}\text{C}(\alpha, \gamma){}^{16}\text{O}$ ,  
 545  ${}^{13}\text{C}(\alpha, n){}^{16}\text{O}$ , and  ${}^{14}\text{N}(\alpha, \gamma){}^{18}\text{O}$ .

546 From the 1990s onwards, YREC has adopted the *J. N.*  
 547 *Bahcall & M. H. Pinsonneault (1992)* approach for hy-  
 548 drogen burning, appropriate for reactions where reso-  
 549 nances do not play a strong role. In this approach, we

550 use the charges, masses, the cross section extrapolated  
 551 to zero energy,  $S_0$ , and its first and second derivatives,  
 552 to directly compute reaction rates. YREC includes elec-  
 553 tron screening of reactions, which includes the interme-  
 554 diate and strong screening cases. However, for general  
 555 use the default code settings recommend the usage of  
 556 weak screening (E. E. Salpeter 1954). The adopted cross  
 557 sections are taken from E. G. Adelberger et al. (2011),  
 558 sometimes referred to as Solar Fusion II. The recently  
 559 published Solar Fusion III rates (B. Acharya et al. 2025)  
 560 are an alternative, and a case using these rates is in-  
 561 cluded in our test suite.

562 Solar neutrino fluxes are computed for all pp and CNO  
 563 neutrino sources, adopting equilibrium abundances for  
 564 all isotopes not explicitly tracked. Neutrino losses are  
 565 subtracted from the effective energy yield of nuclear re-  
 566 actions. YREC uses the N. Itoh et al. (1996) rates for  
 567 continuum neutrino cooling. These are important pro-  
 568 cesses for evolved stars, and they can lead to off-center  
 569 He ignition.

### 2.3.5. *Mixing and Composition Change*

570  
 571 YREC solves for non-equilibrium abundance changes  
 572 due to nuclear reactions using an implicit scheme. For  
 573 convective regions, instantaneous mixing is assumed,  
 574 and the reaction rates are therefore mass-averaged  
 575 across the convection zone. The nuclear reaction rates  
 576 are computed using start-of-timestep values for the  
 577 structure variables. There are options to refine these  
 578 estimates using end-of-timestep values instead. Mi-  
 579 croscopic diffusion and rotationally induced mixing are  
 580 treated in separate, sequential steps.

581 We note that there is a diversity of approaches in the  
 582 literature in how to treat the interaction between com-  
 583 position change and the evolution of the stellar struc-  
 584 ture. At one limit, the P. P. Eggleton (1972) method  
 585 simultaneously solves for composition change and the  
 586 change in the structural variables. The prime author  
 587 of the MESA code built his first evolution code using  
 588 Eggleton as a basis (B. Paxton 2004), and the MESA  
 589 code is built using the same principle. The YREC se-  
 590 quential approach is more common, and faster, but it  
 591 is by no means universal. The same comment applies  
 592 to the choice of where to evaluate the nuclear reaction  
 593 rates in the timestep, or how to treat a mixed convection  
 594 zone whose depth varies during the timestep.

595 In practice, the numerical controls are typically ad-  
 596 justed such that these different approximations yield  
 597 consistent results within the domains where they are  
 598 valid. As an example of a case where these differences  
 599 matter, consider the case of instantaneous mixing in con-  
 600 vection zones. This is usually an excellent approxima-

tion, but it breaks down in interesting domains, such as  
 AGB shell flashes.

### 2.3.6. *Boundary Conditions*

604 The boundary conditions are evaluated at three pairs  
 605 of  $(L, T_{\text{eff}})$  close to the model values (see section 2.2).  
 606 The structure variables (P, T, R, L) of the interior solu-  
 607 tion and their derivatives are matched to the envelope  
 608 solution, and the three different solutions in turn pro-  
 609 vide derivatives as a function of  $(L, T_{\text{eff}})$ .

610 For the atmosphere, the code assumes constant M, R,  
 611 and L, and integrates  $P(T(\tau))$  as a function of optical  
 612 depth  $\tau$  to a fitting point. For the default option, a gray  
 613 atmosphere, this is  $\tau = 2/3$ . Other fitting points can  
 614 be chosen. The integration starts at an arbitrary low  
 615 density and requires a  $T(\tau)$  relationship. Users can also  
 616 use a table look-up from libraries of model atmospheres.  
 617 For the existing tables, an optical depth of  $2/3$  is also  
 618 adopted. The available options are R. Kurucz (1993), F.  
 619 Castelli & R. L. Kurucz (2003), and F. Allard & P. H.  
 620 Hauschildt (1995). The Allard tables are available for  
 621 solar metallicity only, while the other two are available  
 622 for a range of metallicities. If the models go outside  
 623 of the table grid on the hot edge, the table look-up is  
 624 disabled and the code defaults to an Eddington gray  
 625 atmosphere. The run will terminate if the code falls  
 626 outside of the table at high L or low  $T_{\text{eff}}$ .

627 The atmosphere is then used as a starting point for  
 628 an envelope integration, which assumes a constant L.  
 629 The remaining three structure equations are integrated  
 630 down to a fitting point using a Bulirsch-Stoer integration  
 631 scheme with an adaptive step size. The physics used in  
 632 the envelope integration are the same as those used in  
 633 the interiors model. Only the lower boundary condition  
 634 of the envelope integration is retained, and these results  
 635 are output in the last and stored model headers.

### 2.4. *Microscopic Diffusion*

637 In a traditional stellar model, radiative regions are  
 638 strictly stable. In the real world, this is not true. Heavy  
 639 species sink relative to light ones due to gravitational  
 640 settling and thermal diffusion. Large atoms can be  
 641 driven up relative to nuclei and small atoms by radi-  
 642 ation pressure, sometimes referred to as radiative levi-  
 643 tation. Collectively, we refer to these mechanisms as mi-  
 644 croscopic diffusion. The timescale for diffusion is short-  
 645 est in the outer layers of stars, where the density is low,  
 646 and it becomes shorter in stars with thin surface con-  
 647 vection zones. Mass loss can counteract diffusion, and  
 648 diffusion therefore becomes less important on the up-  
 649 per main sequence with the onset of strong radiation  
 650 pressure-driven winds. Microscopic diffusion is there-  
 651 fore particularly important in intermediate mass stars,

between 1.3 and 9  $M_{\odot}$  at solar metallicity. Even in the Sun, however, it impacts surface abundances relative to birth values at the 10% level.

YREC models can include gravitational settling and thermal diffusion. We follow the approach of J. M. Burgers (1969) as implemented by A. A. Thoul et al. (1994). The code includes microscopic diffusion for metals as a group, helium, and the light species  ${}^6\text{Li}$ ,  ${}^7\text{Li}$  and  ${}^9\text{Be}$ . Metals are assumed to all settle at the same rate as fully ionized iron. This is consistent with the approximations used in the opacity tables, which assume a fixed heavy element mixture.

In the limit of very efficient microscopic diffusion, stars will develop layer cake structures in their outer layers; pure H at the top, with a layer of pure He below, followed by zones with progressively heavier species. This causes difficulties for codes in thin surface convection zones due to interpolation issues in opacity and equation of state tables. The opacity for zero H, for example, is quite different from that for even a small trace contribution—see E. Farag et al. (2024) for a discussion of this point. YREC disables microscopic diffusion calculations when the central hydrogen drops below a user-specified threshold (default 0.001), or if the outer point in the model is radiative. In practice, this means that one can choose an outer fitting point deep enough to avoid pathological solutions on the main sequence, which manifest as the code failing due to interpolation errors.

### 2.5. The Structural Effects of Star Spots

Magnetic fields are remarkably hard to model, and thus usually neglected in stellar models. At some level, this omission is reasonable; it is unlikely, for example, that there are core magnetic fields strong enough to compete with gas or radiation pressure, at least in sensible stars. However, there are contexts where magnetic fields cannot be ignored, despite the fondest wishes of theorists. Angular momentum loss by magnetized winds, and magnetic instabilities for rotating stars, can play important roles in models of rotating stars. We discuss magnetic fields in these contexts in Section 2.6, where we address rotation.

Another regime, included in YREC, is the direct impact of star spots on the structure and evolution of cool stars. Stars with modest rotation rates and deep surface convection zones have dynamo-generated magnetic fields. The dynamo produces cool star spots with equipartition-strength magnetic fields. As a consequence, the ambient temperature in un-spotted regions is not the same at the surface averaged effective temperature. Convection is also observed to be inhibited in sunspots, and the same should apply to star spots. As

a consequence, star spots can be consequential for stellar structure, as originally pointed out in H. C. Spruit (1982).

Although the solar sunspot filling factor is small, of order 0.1 percent, there are stars where spots are far more important. In active main sequence stars, the filling factor of spots can reach levels of order 25 percent; much higher filling factors, 70 percent or more, are seen in pre-main sequence stars (L. Cao & K. G. Stassun 2025).

YREC accounts for both flux blocking and a modified surface boundary condition (G. Somers & M. H. Pinsonneault 2015). Star spots can induce radius inflation and impact pre-MS lithium depletion. In fully convective stars the luminosity can also be substantively altered. The star spot filling factor and temperature contrast is specified at the start of the run and held fixed, which allows users to test for the structural effects of star spots.

### 2.6. Rotation

Rotation is a fundamental stellar property. Stars are born with a wide range of rotation rates. As a consequence stars of the same birth mass, composition, and age can have very different properties. Rotation changes stellar structure, altering the surface appearance and changing fundamental properties such as lifetimes. It induces mixing in otherwise stable regions, which can be consequential for chemical evolution. Rotation can also be an age indicator, making it a key diagnostic tool for stellar populations.

Rotation is also challenging to model, as it is intertwined with frontier challenges in stellar theory. The interplay of rotation and magneto-convection generates stellar dynamos. Magnetized winds drive angular momentum loss in some regimes, but not others. In radiative regions magnetic fields, waves, and hydrodynamic mechanisms can transport angular momentum, and induce mixing of chemical species, to varying degrees. In convective regions the overturn timescale is short compared to the evolutionary timescale, so a rotation profile will be enforced by the interplay of rotation and turbulence. The natural timescales of both rotation and turbulence vary by orders of magnitude, however, so there is no reason to expect all stars to have some universal profile. It is thus unsurprising that we lack a consensus model for stellar rotation. YREC includes a variety of options, which includes the freedom to explore quite different underlying physical models for the angular momentum evolution of stars.

There is a global correspondence between the MESA and YREC codes in terms of the treatment of rotation. The MESA method is based on the approach of A. Heger

et al. (2000), which in turn used M. H. Pinsonneault et al. (1989) as a starting point. Both papers, in turn, broadly follow the approach of A. S. Endal & S. Sofia (1976). In detail, however, the different codes have distinctive approaches. Due to the contributions of many people, over the years these models have evolved and mutated. We document this natural process of rejuvenation, particularly in the treatment of angular momentum transport, loss, and mixing, here.

### 2.6.1. *The Initial Conditions for Rotation*

The natural initial condition for rotation is in the star formation phase, which in turn divides into two phases. Stellar modeling typically begins at the end of the hydrodynamic collapse phase. At this phase the star is assumed to have reached its final mass and hydrostatic equilibrium is a good approximation. In YREC, this would correspond to the deuterium burning birthline. Spherical models can be converted into uniformly rotating ones at a user-specified rate for any model, but the birthline is a physically motivated state, at least for low mass stars.

Starting models much further up the Hayashi track exist, but these do not correspond to real stars. This is an important point for initializing rotation. If a user initializes rotation in an artificially large starting model, angular momentum conservation will transform a reasonable sounding guess into an unreasonable outcome.

During the hydrostatic pre-MS, star-disk coupling can occur (A. Koenigl 1991; F. Shu et al. 1994). It is also necessary to explain the evolution of rotation in young stars. As a result, YREC also includes the option to enforce star-disk coupling for a user-specified timescale at a user-specified rate.

For massive stars, it is less clear that the deuterium burning birthline is a true representation of the starting state (F. Palla & S. W. Stahler 1991). Above a critical mass, of order 5 to 8 solar masses, accreting models develop radiative cores and are born close to the main sequence. For higher mass models, an initial rotation state on the main sequence may therefore be closer to being physically reasonable.

### 2.6.2. *The Structural Effects of Rotation*

Rotation induces a departure from spherical symmetry which can be significant. The net effect is a reduction in the effective gravitational force, which makes rotating stars behave for most purposes as if they had a lower effective mass. However, in detail the impact depends on the angular momentum distribution. The general case would require a two or three dimensional model. Fortunately, there are reasonable families of models that can

be captured using a pseudo-2D angular velocity distribution. In this case, the structural effects of rotation can be computed by measuring the shape of equipotential surfaces as a function of mass. The impact of rotation on the structure can then be included with correction terms applied to the spherically symmetric equations of stellar structure.

YREC uses a modified version of the R. Kippenhahn & H. C. Thomas (1970) technique. See M. H. Pinsonneault et al. (1989) for a discussion of the differences. This method assumes uniform rotation on equipotential surfaces. The code then solves for the shape of these surfaces, including contributions from the quadrupole term in the potential, and solves a modified set of stellar structure equations.

### 2.6.3. *Rotation in Convective Regions*

YREC assumes that a rotation law is enforced in convective regions on a short timescale. The code permits rotation profiles of the form  $\omega \sim R^\alpha$ ,  $-2 \leq \alpha \leq 0$ . There are also options to couple the rotation profile to adjacent radiative regions - for example, to match the rate at the base of a convective region to the rate in the radiative zone below it. These options have a significant impact for the rotational evolution of post-main sequence stars, where observational data suggests that core rotation rates can be tens to hundreds of times faster than the envelope rotation rates (B. Mosser et al. 2012; C. Gehan et al. 2018, e.g.), with important implications for internal angular momentum transport (J. Tayar & M. H. Pinsonneault 2013; J. Fuller et al. 2019). Both the surface and internal rotation of core He burning stars are important tests of angular momentum loss (J. Tayar & M. H. Pinsonneault 2018). Rotation and magnetism may also affect mass loss on the red giant branch (Y. Li 2025).

YREC does not include differential rotation with latitude. For sunlike stars, the average rotation with depth is nearly constant, even if there is latitudinal differential rotation. We therefore recommend that users impose rigid rotation for pre-MS and MS cases. However, they should keep in mind that imposing a uniform rotation rate in convection zones may be a poor approximation for luminous giants. It is difficult to explain the survival of rotation in blue core He-burning stars without strong differential rotation in luminous red giants (M. H. Pinsonneault et al. 1991; A. Sills & M. H. Pinsonneault 2000). This differential rotation can also impact the interpretation of red giant core rotation rates (Y. Kissin & C. Thompson 2015) and core He burning stars (J. Tayar & M. H. Pinsonneault 2018).

#### 2.6.4. Angular Momentum Loss From Magnetized Winds

Users may choose between braking laws that describe the angular momentum loss via magnetized winds. Magnetic braking is the consequence of the interaction of mass loss and magnetic fields. As both are frontier fields in stellar physics, the description of their interaction is unsurprisingly burdened with many assumptions.

Broadly, the rate of angular momentum (AM) loss is set by a combination of the mass loss rate ( $\dot{M}$ ), the rotation rate ( $\Omega$ ), and the effective magnetic lever arm, parameterized as the Alfvén radius ( $R_A$ ). The Alfvén radius itself generally depends on  $\dot{M}$ , stellar magnetic field ( $B$ ), and wind driving physics. The overall field configuration (dipole, quadrupole, etc.) affects how strongly the torque scales with the magnetic lever arm. YREC has three built-in options for the braking law that differ in their assumptions about each of these ingredients.

The first is a modified version of the [S. D. Kawaler \(1988\)](#) prescription proposed by [A. Krishnamurthi et al. \(1997\)](#). The Kawaler braking law assumes that a dipole magnetic field scales as  $BR^2 \sim \Omega$ . The torque for slow rotators therefore scales as  $\Omega^3$ . [A. Krishnamurthi et al. \(1997\)](#) added a Rossby-scaled magnetic saturation threshold to the standard form of the Kawaler law, beyond which the AM loss scales more weakly with rotation rate. This saturation is a ubiquitous feature of magnetic proxies in stars: they take maximal but constant values at sufficiently rapid rotation and low Rossby number  $Ro$  (the rotation period divided by the convective overturn timescale).

The second form is the loss law from [J. L. van Saders & M. H. Pinsonneault \(2013\)](#), which uses a parametrized model of the steady-state MHD wind solutions from [S. Matt & R. E. Pudritz \(2008\)](#) and [S. P. Matt et al. \(2012\)](#). The magnetic field is scaled assuming a linear (Rossby-scaled) dynamo and pressure equipartition at the photosphere:  $B \sim Ro^2 P_{phot}^{1/2}$ . Mass loss is scaled as  $\dot{M} \sim L_X \sim (\omega\tau_{cz})^2$  following the observed correlation of  $\dot{M}$  with X-ray luminosity ([B. E. Wood et al. 2005](#)) and scaling of the X-ray luminosity with Rossby number ([N. Pizzolato et al. 2003](#)). The liberal use of Rossby scalings causes the torque to be significantly weaker in stars with thin convective envelopes than that in a Kawaler prescription, which better matches the actual rotational behavior of stars near the Kraft break.

Finally, users can enable a “custom” mode where they can manually set the exponents that describe the dynamo scalings, Rossby scalings, and effective magnetic index.

Although the braking utilizes an inferred  $\dot{M}$ , this  $\dot{M}$  is not self-consistently incorporated into the structural

evolution. All prescriptions available in YREC allow the user to specify a disk-locking rotation period and duration. Magnetic saturation of the torque is accounted for, with the saturation threshold set by the user. Because the field strengths, mass loss rates, and overall torque magnitudes are not known from first principles, practical use always involves the adjustment of a scale factor to calibrate the magnitude of the torque against benchmark observations. Sample cases of such a calibration are discussed in [Section 3.4.4](#).

#### 2.6.5. Angular Momentum Transport and Rotationally Induced Mixing in Radiative Regions

The original YREC code included angular momentum transport and the associated mixing via hydrodynamic mechanisms. YREC solves diffusion equations for the transport of angular momentum and the associated rotationally induced mixing. Instantaneous mixing is assumed in convective regions. Angular momentum loss is induced in the surface convection zone as a surface boundary condition.

The secular mechanisms included are meridional circulation, the secular shear instability, and the Goldreich-Schubert-Fricke instability. See [G. Somers & M. H. Pinsonneault \(2016\)](#) for a recent summary of the assumed input physics. This differs from the [M. H. Pinsonneault et al. \(1989\)](#) approach in that the characteristic length scale for the instabilities is the radius, rather than the velocity scale height. The meridional circulation treatment is also modeled on the [J. P. Zahn \(1992\)](#) approach, neglecting terms of order the second or higher derivative of the angular velocity but including the quadrupole component of the potential. The hyper-diffusion terms in the [J. P. Zahn \(1992\)](#) method require a very different numerical treatment than the one used here. This expression is broadly similar to others in the literature. We include the reduction in the effective meridional circulation velocity induced by mean molecular weight ( $\mu$ ) gradients following the approach of [A. Maeder & J.-P. Zahn \(1998\)](#). We also adopt the [A. Maeder \(1997\)](#) approach for the secular shear instability. The code permits different treatments of the inhibiting effect of mean molecular weight gradients on angular momentum transport and the associated mixing.

A central result from [M. H. Pinsonneault et al. \(1989\)](#) was that the efficiency of angular momentum transport must be much higher than that of the associated mixing. This was placed on a physical basis by [B. Chaboyer & J. P. Zahn \(1992\)](#), who argued that the turbulence induced by rotation is much more effective in the horizontal direction than in the vertical direction. The former can exchange angular momentum, while mixing is only induced in the vertical direction. YREC treats this as

957 a free parameter,  $f_c$ , which can be used to adjust the  
 958 efficiency of rotationally induced mixing. This differs  
 959 from the J. P. Zahn (1992) approach, where this is a  
 960 fixed parameter of the model. In our view, the assump-  
 961 tions embedded in the approach make a free parameter  
 962 an appropriate choice.

963 Wave-driven transport is not supported in the pub-  
 964 lic release, but we intend to add in in later releases.  
 965 Magnetic transport is modeled under two different treat-  
 966 ments. Strong magnetic coupling, as that predicted  
 967 from the Taylor-Spruit mechanism, is effectively equiv-  
 968 alent to uniform rotation on the lower main sequence.  
 969 We therefore include an option to simply assume uni-  
 970 form rotation at all times, which mimics the effects of  
 971 strong coupling. We also include an option for a diffusive  
 972 magnetic transport term (G. Somers & M. H. Pinson-  
 973 neault 2016) that does not induce mixing; this can be  
 974 adjusted to reproduce, for example, the core-envelope  
 975 coupling timescale inferred from the spin down of low  
 976 mass stars, or the core to surface rotation contrast in  
 977 the Sun. This approach is now used in other codes to  
 978 mimic magnetic transport (P. Eggenberger et al. 2019).  
 979 The code also allows users to couple core rotation pro-  
 980 files to differentially rotating convective envelopes.

### 981 3. SAMPLE APPLICATIONS AND LITERATURE 982 COMPARISONS

983 The life cycles of stars are wondrously complex.  
 984 YREC is a flexible tool, which can simulate many scen-  
 985 arios. It is important to understand the precision, accu-  
 986 racy, and domain of validity for the code. We there-  
 987 fore present different physical scenarios as well as differ-  
 988 ent mass, composition, and evolutionary state domains  
 989 for the code. We begin by laying out the properties of  
 990 our “base case” models, and follow by discussing mod-  
 991 els with different input physics or initial conditions. We  
 992 discuss scripts to run sets of models, and tools to extract  
 993 information from the code, in Section 4.

#### 994 3.1. The Base Case and Numerical Precision Settings

995 Our base case adopts the physical assumptions listed  
 996 in Table 1, and—coupled with the test suite cases—  
 997 represents a reasonable starting point for users devel-  
 998 oping their own input namelists. We use a solar calibra-  
 999 tion for the convective mixing length  $\alpha$  and the birth  
 1000 solar helium  $Y_{\odot,init}$ . For models with non-solar metal  
 1001 content, we use the difference between the solar and Big  
 1002 Bang nucleosynthetic Y abundance (Planck Collabora-  
 1003 tion et al. 2020) to define the helium to metal enrichment  
 1004 ratio  $\Delta Y/\Delta Z$ . We note that other codes and isochrones  
 1005 make different assumptions about this ratio, which also  
 1006 depends on the choice of input physics. Our adopted

**Table 1.** Summary of Input Physics for the Base Model Grid

Parameter	YREC	
Atmosphere	Gray	
Convective Core Over- shoot	Lesser of Step: $0.2H_p$ , 0.15 core size	
Diffusion	None	
Equation of State	OPAL <sup>a,b</sup> + SCV <sup>c</sup>	
High-Temperature Opacities	OP <sup>d</sup>	
Low-Temperature Opac- ities	J. W. Ferguson et al. (2005)	
Mixing Length	1.707873	
Nuclear Reaction Rates	E. G. Adelberger et al. (2011)	
Rotation	None	
Weak Screening	E. E. Salpeter (1954)	
Mixture and Solar Z/X	N. Grevesse & A. J. Sauval (1998)	
Solar X	0.719600	
Solar Y	0.263908	
Solar Z	0.016492	
$\Delta Y/\Delta Z$	0.951249	
Surface $(Z/X)_{\odot}$	0.02292	
	Metallicity Map	
[Fe/H]	X	Z
-1.0	0.750950	0.001721
-0.5	0.743176	0.005386
+0.5	0.654060	0.047401

<sup>a</sup> F. J. Rogers et al. (1996); <sup>b</sup> F. J. Rogers & A. Nayfonov (2002); <sup>c</sup> D. Saumon et al. (1995); <sup>d</sup> N. R. Badnell et al. (2005)

1007 birth hydrogen and metal abundances (X and Z) as a  
 1008 function of [Fe/H] are also given in Table 1, assuming a  
 1009 GS98 abundance pattern.

1010 The numerical choices in our models are customized to  
 1011 the requirements for different use cases. In some cases,  
 1012 the tolerances for numerical precision had to be relaxed  
 1013 to obtain a solution. We note the domains where the rec-  
 1014 ommended numerical controls differ significantly from  
 1015 the general case in the discussion below.

1016 Our spatial resolution, temporal resolution, and con-  
 1017 vergence criteria for all cases are defined such that the  
 1018 lifetime of each phase is precise at the 1 percent level.  
 1019 Solar models require  $L = L_{\odot}$ ,  $3.827 \times 10^{33}$  erg  $s^{-1}$ , and  
 1020  $R = R_{\odot}$ ,  $6.958 \times 10^{10}$  cm, at the solar age, 4.568 Gyr,  
 1021 within one part in  $10^{-4}$ . The surface  $\frac{Z}{X}$  is required to be  
 1022 within 1 percent of the current solar surface  $\frac{Z_{\odot}}{X_{\odot}}$ , 0.02292  
 1023 for the N. Grevesse & A. J. Sauval (1998) mixture. We  
 1024 also require that neutrino fluxes were precise at the 1  
 1025 percent level. Solar models including rotation also re-  
 1026 produce the solar equatorial rotation rate (25.4 days)  
 1027 and the difference between the meteoritic and the cur-

1028 rent solar surface  ${}^7\text{Li}$  abundance (2.3 dex) within 1 per- 1079  
 1029 cent. These are achieved by varying the scaling constant 1080  
 1030 in the magnetized wind and the efficiency of mixing rela- 1081  
 1031 tive to angular momentum transport, respectively. 1082

1032 The inner fitting point is set at  $10^{-4}M_{\text{tot}}$ . For typical 1083  
 1033 main sequence conditions, this places the inner fitting 1084  
 1034 point close to  $0.01R_{\text{tot}}$ . Solar models, and the mod- 1085  
 1035 els showing diffusion and mixing to the terminal-age 1086  
 1036 main sequence (TAMS), have the outer fitting point set 1087  
 1037 at  $10^{-7}M_{\odot}$  below the surface. For solar models, this 1088  
 1038 places the outer fitting point close to  $0.99R_{\text{tot}}$ . Gen- 1089  
 1039 eral purpose stellar evolution runs have the outer fitting 1090  
 1040 point set at  $10^{-4}M_{\text{tot}}$  below the surface, which on the 1091  
 1041 main sequence corresponds to roughly  $0.95R_{\text{tot}}$ . In some 1092  
 1042 cases the fitting point location is adjusted differently to 1093  
 1043 achieve proper numerical results. We also evolved some 1094  
 1044 models in stages, with somewhat different numerical tol- 1095  
 1045 erances in different regimes. 1096

1046 We use Hayashi track seed models to generate mod- 1097  
 1047 els at the beginning of the deuterium burning birthline, 1098  
 1048 defined as the location where 1 percent of the birth deu- 1099  
 1049 terium has been burned. These models are the start- 1100  
 1050 ing point for the large majority of the cases that we 1101  
 1051 present here. We provide libraries of starting models 1102  
 1052 with a range of masses and metallicities. In addition 1103  
 1053 to the GS98 mix, we also provide models with the M. 1104  
 1054 Asplund et al. (2021), E. Magg et al. (2022), and K. Lod- 1105  
 1055 ders et al. (2025) heavy element mixtures. The default 1106  
 1056 YREC models use the K. Lodders (2021) isotopic ratios 1107  
 1057 for CNO and the light element abundances D,  ${}^3\text{He}$ ,  ${}^6\text{Li}$ , 1108  
 1058  ${}^7\text{Li}$  and  ${}^9\text{Be}$ . 1109

### 1059 3.1.1. Run-speed comparison with MESA

1060 We perform rough speed comparisons between MESA 1110  
 1061 and YREC for low mass stars. For YREC, we run a 1111  
 1062 solar mass, solar metallicity model up to the tip of the 1112  
 1063 red giant branch using the same numerical and physics 1113  
 1064 settings as the base model grid in Section 3.3. For 1114  
 1065 the MESA comparison model, we use MESA version 1115  
 1066 r24.08.1 with most inlist values left at their default 1116  
 1067 settings except those required to match the input physics 1117  
 1068 choices of the YREC model; an example inlist is pro- 1118  
 1069 vided in the sample cases. We adjusted the global mesh 1119  
 1070 variable `mesh_delta_coeff` in MESA for each phase of 1120  
 1071 evolution to roughly match the number of mesh points 1121  
 1072 in the YREC models. The average number of shells 1122  
 1073 across each phase of evolution we consider is within 3% 1123  
 1074 for both codes. Models were run on a single thread 1124  
 1075 to mimic YREC execution and to make the compari- 1125  
 1076 son more hardware agnostic, although in practice users 1126  
 1077 of MESA would utilize all available cores on their ma- 1127  
 1078 chines. From pre-main sequence initialization to the 1128

ZAMS, YREC is roughly 8 times faster than MESA at 1079  
 similar mesh resolution; from the ZAMS to TAMS it is 1080  
 $\sim 2\times$  faster; and from the TAMS to TRGB, YREC is 1081  
 $\sim 6\times$  faster. “Default” YREC runs with fewer shells 1082  
 than default MESA: the mesh was made sparser in each 1083  
 of these evolutionary phases to match YREC. In prac- 1084  
 tice, the speed gains may therefore be even greater when 1085  
 models are tuned to achieve the numerical precision re- 1086  
 quired for the science question at hand. Users can use 1087  
 the simplified Yale EoS for another factor of  $\sim 9$  speed 1088  
 gain over the base case pre-MS to TRGB run, which al- 1089  
 lows for rapid testing (54 to 72 times faster execution 1090  
 time than MESA) with only modest sacrifices in physical 1091  
 accuracy. 1092

### 1093 3.2. The Test Suite

1094 A test suite is provided to demonstrate YREC’s ca- 1095  
 1096 pabilities and provide for a baseline reference to aid in 1097  
 regression testing during development. It is not yet com- 1098  
 prehensive, but is being added to regularly and our aim 1099  
 is to provide full coverage of use cases and model types 1100  
 that YREC supports. Information on setting up and 1101  
 running all or a portion of the pre-defined test suite or 1102  
 adding one’s own test cases may be found in the ‘exam- 1103  
 ples’ subdirectory of the project file tree. 1104

1105 Our test suite is designed to test physical scenarios 1106  
 in current use, and to include a wide range of physi- 1107  
 cal conditions. We recommend that users who modify 1108  
 the code validate their changes against the standard li- 1109  
 brary of test suite results. Our physical assumptions, 1110  
 the masses, birth compositions, and mixing lengths are 1111  
 listed in Table 2. We will present the results from the 1112  
 test suite and other models in subsequent sections. The 1113  
 namelists also provide examples of how users can cus- 1114  
 tomize the code for different use cases. 1115

### 1116 3.3. The Base Case Model Grid

1117 We adopted the test suite templates to run our base 1118  
 1119 grid of models, using the same physical assumptions de- 1120  
 1121 scribed in Section 3.1. The grid spans masses from 0.3 1122  
 1123 to  $8.0 M_{\odot}$  in increments of  $0.1 M_{\odot}$ , and metallicities of 1124  
 $[\text{Fe}/\text{H}] = -1.0, -0.5, 0.0,$  and  $+0.5$ . Models with ini- 1125  
 tial masses below  $2.1 M_{\odot}$  were evolved to the tip of the 1126  
 red giant branch, while the remainder were evolved to 1127  
 the onset of core helium burning. We list in Table 1 the 1128  
 physical assumptions made to generate the grid. We use 1129  
 this grid to define the regime where YREC is expected 1130  
 to produce reliable results for stellar evolution and to 1131  
 demonstrate its predictions. 1132

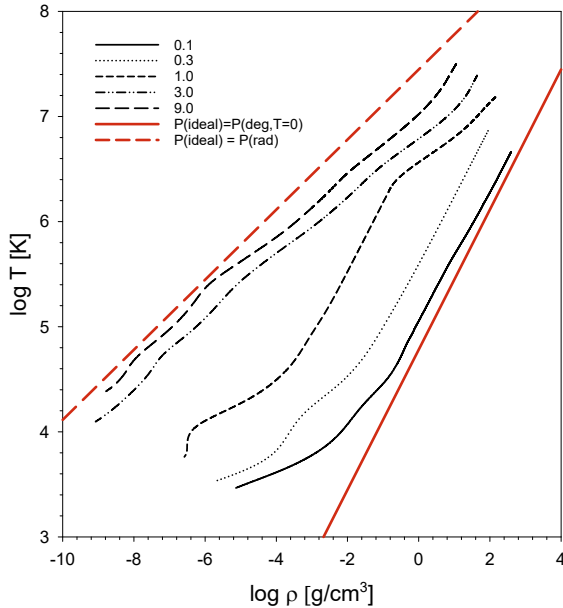
#### 1133 3.3.1. From the Deuterium Birthline to Core He ignition.

1134 Snapshots of the structure of zero-age main sequence 1135  
 1136 models is a logical starting point. In Figure 2, we show 1137  
 1138

**Table 2.** Cases in the YREC test suite. Column 1 contains the name, or names, of the cases; Column 2 is the type, and Column 3 is a brief description. Solar type runs are calibrated solar models, designed to showcase common use cases. Zero-age main sequence (ZAMS) runs, defined as where core H drops 0.005 below the birth value, are for the lower main sequence, and highlight the impact of different choices of surface boundary conditions and equations of state. Brown dwarf runs, taken to 15 Gyr, test code performance below the H burning limit. Terminal-age main sequence (TAMS) runs are taken to core H exhaustion, defined as when the central H drops to 0.0001. TAMS runs show the effects of diffusion, rotation, and rotationally induced mixing below and just above the Kraft break. Evolution runs are longer sets chained together. The  $0.3 M_{\odot}$  case runs first to the ZAMS and next to the TAMS. The  $1.0 M_{\odot}$  case runs first to the TAMS, then to He ignition in a degenerate flash. It then restarts at the zero-age horizontal branch (ZAHB), and ends at core He exhaustion (TAHB). The  $3.0$  and  $9.0 M_{\odot}$  cases are evolved to the TAMS, the ZAHB, and the TAHB in three runs.

Case Name	Type	Descriptions
Test solarGS98 base	Solar	Base case.
Test solarGS98 dif	Solar	Diffusion included.
Test solarAAG21 dif	Solar	AAG21 mix, diffusion included.
Test solarM22M dif	Solar	Magg22 mix, diffusion included.
Test solarGS98 dif rot	Solar	Diffusion, rotation, and rotational mixing included.
Test solarGS98 dif rot fast	Solar	As the dif rot case, but rapid birth rotation.
Test solarGS98 dif rot solid	Solar	As the dif rot case, but solid body rotation.
Test solarGS98 kurucz	Solar	Kurucz atmosphere SBC.
Test solarGS98 SF3	Solar	Solar fusion III nuclear reaction rates.
Test solarGS98 yaleeos	Solar	Yale EoS.
m0p03feh+0.0GS98 allard 15 Gyr	Brown Dwarf	$0.03M_{\odot}$ , Allard SBC, to 15 Gyr.
m0p05feh+0.0GS98 allard 15 Gyr	Brown Dwarf	$0.05M_{\odot}$ , Allard SBC, to 15 Gyr.
m0p08feh+0.0GS98 allard 15 Gyr	Brown Dwarf	$0.08M_{\odot}$ , Allard SBC, to 15 Gyr.
m0p1feh+0.0GS98 allard ZAMS	ZAMS	$0.1M_{\odot}$ , Allard SBC.
m0p1feh+0.0GS98 base ZAMS	ZAMS	$0.1M_{\odot}$ , base case.
m0p1feh+0.0GS98 spot25 ZAMS	ZAMS	$0.1M_{\odot}$ , fspot 0.25 case.
m0p3feh+0.0GS98 allard ZAMS	ZAMS	$0.3M_{\odot}$ , Allard SBC.
m0p3feh+0.0GS98 spot25 ZAMS	ZAMS	$0.3M_{\odot}$ , fspot 0.25 case.
m0p3feh+0.0GS98 scveos ZAMS	ZAMS	$0.3M_{\odot}$ , SCV EoS.
m0p3feh+0.0GS98 yaleeos ZAMS	ZAMS	$0.3M_{\odot}$ , Yale EoS.
m1p0feh+0.0GS98 spot25 ZAMS	ZAMS	$1.0M_{\odot}$ , fspot 0.25 case.
m1p0feh+0.0GS98 dif TAMS	TAMS	$1.0M_{\odot}$ , diffusion included, evolved to TAMS.
m1p4feh+0.0GS98 dif TAMS	TAMS	$1.4M_{\odot}$ , diffusion included, evolved to TAMS.
m0p3feh+0.0GS98 base ZAMS, TAMS	Evolution	$0.3M_{\odot}$ , base case evolved to ZAMS, TAMS.
m1p0feh+0.0GS98 base TAMS, HeIgnite, TAHB	Evolution	$1.0M_{\odot}$ , base cases, evolved to the TAMS, He ignition, and from core He burning to core He exhaustion.
m3p0feh+0.0GS98 base TAMS, ZAHB, TAHB	Evolution	$3.0M_{\odot}$ , base cases, evolved to the TAMS, ZAHB and TAHB.
m9p0feh+0.0GS98 base TAMS, ZAHB, TAHB	Evolution	$9.0M_{\odot}$ , base cases, evolved to the TAMS, ZAHB and TAHB.
m9p0feh+0.0GS98 yaleeos TAMS, ZAHB, TAHB	Evolution	$9.0M_{\odot}$ , base cases with Yale EoS, evolved to the TAMS, ZAHB and TAHB.
Solar_m1p0feh+0p0_GN93_TAHB	TAHB	$1.0M_{\odot}$ $Z=0.019$ ZAHB evolved to the TAHB, with GN93 opacity tables.
Solar_m0p7feh+0p75_GN93_TAHB	TAHB	$0.7M_{\odot}$ $Z=0.003$ ZAHB evolved to the TAHB, with GN93 opacity tables.
Solar_m0p95feh+0p0_GN93.TAHB	TAHB	$0.95M_{\odot}$ $Z=0.019$ ZAHB evolved to the TAHB via ZAHB seed envelope mass re-scaling, with GN93 opacity tables.

1129 the internal structure of representative ZAMS models  
 1130 in the  $\rho - T$  plane, an important indicator of the pre-  
 1131 dominant pressure sources from the equation of state.  
 1132 We also show the boundaries where radiation pressure  
 1133 (upper left) and degeneracy pressure (lower right) domi-  
 1134 nate.

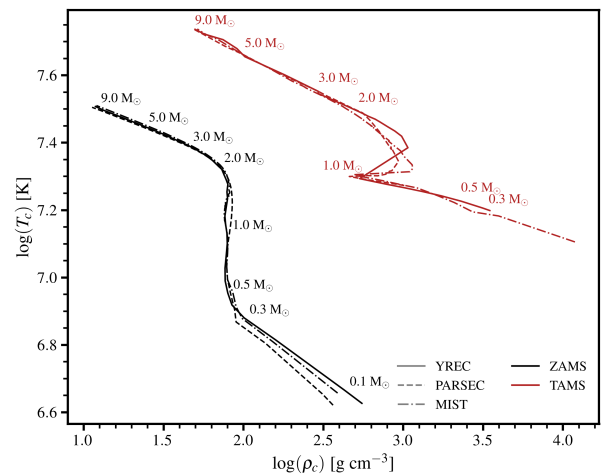


**Figure 2.** The locus in the  $\rho - T$  plane of 0.1(lower right), 0.3, 1, 3 and 9  $M_{\odot}$  models (upper left) at the zero-age main sequence, defined here as the point where the central hydrogen mass fraction is 0.005 lower than the birth value. These are solar metallicity models using the base model physics. The domains where radiation pressure equals gas pressure, and where zero-temperature degeneracy pressure equals gas pressure, are indicated by the respective red lines.

1135 The central temperature and density for models as  
 1136 a function of mass is also an interesting diagnostic of  
 1137 stellar structure. We present our models at the start  
 1138 (ZAMS) and end (TAMS) of the main sequence, compared  
 1139 with PARSEC and MIST, in Figure 3. The upper  
 1140 MS has a shallow slope in central T as a function of mass  
 1141 due to the strong temperature dependence of the CNO  
 1142 cycle; in lower mass stars the pp chain is more impor-  
 1143 tant and the slope is steeper there. The steep change in  
 1144 central density close to solar mass reflects the transition  
 1145 from radiative to convective envelopes. Different codes  
 1146 are very close above 0.5  $M_{\odot}$  on the ZAMS, with the dif-  
 1147 ferences at lower mass reflecting the importance of the  
 1148 equation of state and surface boundary conditions for  
 1149 these stars. Although both  $\rho$  and T increase on the MS,  
 1150 the increases are much larger for  $\rho$ . Physical conditions

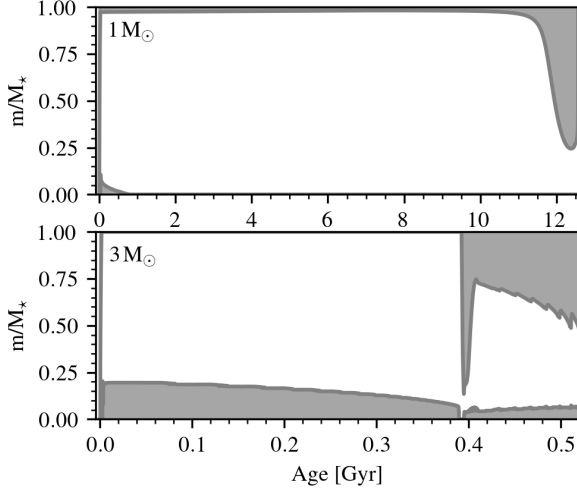
1151 can change rapidly after the MS turnoff, which accounts  
 1152 for some of the differences seen between 1 and 2  $M_{\odot}$ .

1153 The depth of core and surface convection zones are  
 1154 another interesting diagnostic of stellar evolution, fre-  
 1155 quently displayed as a function of time in Kippenhahn  
 1156 diagrams. We show the histories of 1 and 3  $M_{\odot}$  models  
 1157 in Figure 4. The 1  $M_{\odot}$  model has a shallow (in mass)  
 1158 convective envelope on the MS, develops a deep surface  
 1159 convection zone during an extended red giant branch  
 1160 phase, and has a convective core during the core He-  
 1161 burning phase. The 3  $M_{\odot}$  model has a convective core  
 1162 that gradually shrinks during the MS phase, followed by  
 1163 a brief shell H-burning red giant phase. The core He-  
 1164 burning phase is a larger fraction of the lifetime in this  
 1165 model than in the lower mass one.

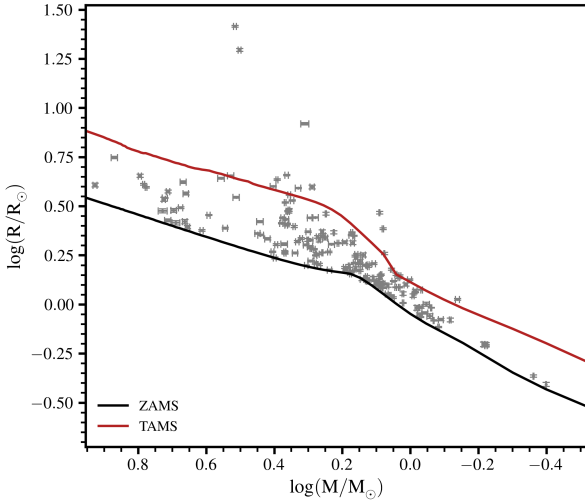


**Figure 3.** Central temperature as a function of central density at ZAMS (black) and TAMS (red) locations. Comparisons are shown between YREC, MIST, and PARSEC evolution tracks for solar metallicity (YREC,  $Z = 0.016492$ ; MIST,  $Z = 0.0142$ ) and  $Z = 0.017$  (PARSEC).

1166 Figure 5 shows the stellar radius as a function of mass,  
 1167 with the black and red lines corresponding to the ZAMS  
 1168 and TAMS locations. The lower envelope is in excellent  
 1169 agreement with fundamental data from eclipsing binary  
 1170 stars (G. Torres et al. 2010). The apparent agreement  
 1171 is deceptive for the lower MS, where the TAMS greatly  
 1172 exceeds the age of the Universe. We discuss the “radius  
 1173 inflation” problem in Section 3.4.5.  
 1174 In Figure 6, we show the evolutionary tracks for solar  
 1175 metallicity models at 0.3, 1.0, 3.0, and 9.0  $M_{\odot}$ . The  
 1176 main sequence band is superposed for reference. The  
 1177 lower branch for each mass track is the pre-MS phase,  
 1178 defined at deuterium ignition. This definition breaks  
 1179 down for the more massive stars, where a physical model  
 1180 would predict a starting point closer to the MS (F. Palla  
 1181 & S. W. Stahler 1991; H. W. Yorke & C. Sonnhalter



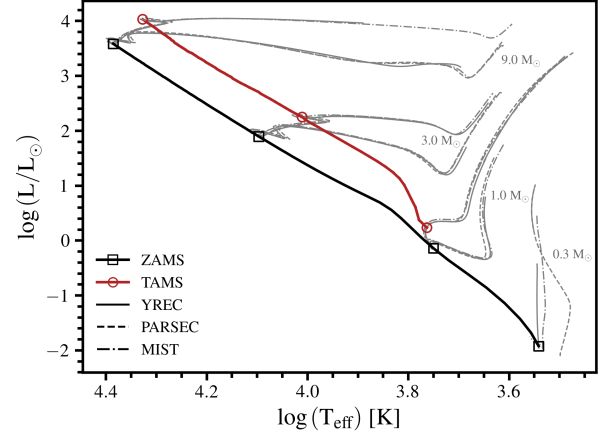
**Figure 4.** Kippenhahn diagram showing the evolution of convective boundaries for the  $1M_{\odot}$  (top panel) and  $3M_{\odot}$  (bottom panel) models. Shaded gray regions indicate convective zones shown as a function of age and fractional mass coordinate.



**Figure 5.** Radius as a function of mass for tracks  $0.3 - 9.0 M_{\odot}$  at ZAMS and TAMS. [G. Torres et al. \(2010\)](#) masses and radii of eclipsing binaries plotted for reference.

2002). A transient loop above the MS, due to non-equilibrium CN burning, is clearly seen in the higher mass tracks, as is the sudden adjustment at the end of the MS phase from H depletion in a large convective core. The  $1$  and  $3 M_{\odot}$  cases are similar in  $L$  during the core He-burning phase, while the  $9 M_{\odot}$  case is significantly brighter due to a larger H-exhausted core on the MS.

The non-rotating models from the MIST ([J. Choi et al. 2016](#)) and PARSEC ( $v1.2$  and  $v2.0$ , [A. Bressan et al.](#)



**Figure 6.** Luminosity as a function of temperature for the solar metallicity models. The solid black and red lines show the location of ZAMS and TAMS, respectively. We show a comparison of evolutionary tracks from YREC (solid), PARSEC (dashed), and MIST (dash-dot) model grids for solar metallicity and  $Z = 0.017$  for PARSEC. Tracks are shown for masses of  $0.3, 1.0, 3.0,$  and  $9.0 M_{\odot}$ , as available in each model library.  $1.0, 3.0,$  and  $9.0 M_{\odot}$  tracks are ended at He core ignition.

1192 [2012; Y. Chen et al. 2014; Y. Chen et al. 2015; X. Fu](#)  
 1193 [et al. 2018 C. T. Nguyen et al. 2022a; G. Costa et al.](#)  
 1194 [2025](#)) libraries are overlaid for comparison with YREC  
 1195 <sup>13</sup>. The tracks for different codes are broadly similar at  
 1196 solar mass and above. Tracks for the lowest mass model  
 1197 ( $0.3 M_{\odot}$ ) are quite different, reflecting the importance of  
 1198 the equation of state and surface boundary conditions.  
 1199 See Section 3.4.2 for a further discussion of this point.

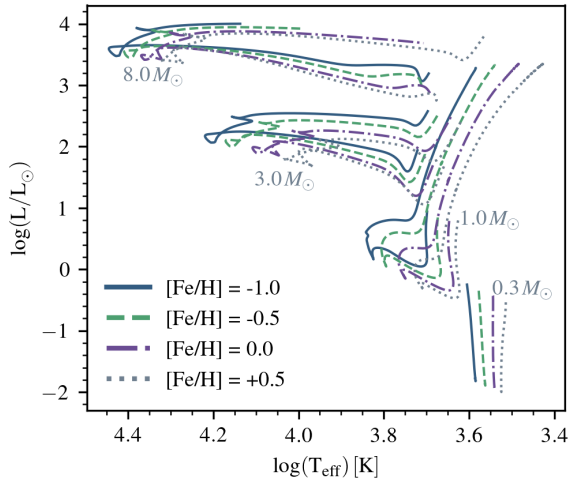
1200 Table 3 provides a summary of stellar parameters at  
 1201 TAMS for the  $1M_{\odot}$  tracks, including age, luminosity, effec-  
 1202 tive temperature, radius, central density, and central  
 1203 temperature. The TAMS location is defined as the point  
 1204 where the central H abundance drops to  $0.0001$ . Both  
 1205 MIST and PARSEC models show close agreement with  
 1206 our base grid in central temperature, effective tempera-  
 1207 ture, and radius, each agreeing within 3%. The MIST  
 1208 models exhibit slightly larger offsets in luminosity and  
 1209 age (6 and 10%, respectively), while the PARSEC mod-  
 1210 els differ primarily in central density and age (6% and  
 1211 7%). These discrepancies can be partially attributed to  
 1212 differences in the precise definition of the TAMS point  
 1213 in each grid and variations in the adopted input physics.  
 1214 For instance, the MIST models use the [R. Kurucz \(1993\)](#)

<sup>13</sup> The PARSEC library provides only pre-MS to ZAMS for  $0.3 M_{\odot}$  and does not include tracks at its defined solar metallicity, where  $Z_{\odot} = 0.01524$  and  $Z_{\text{initial}} = 0.01774$  ([A. Bressan et al. 2012; Y. Chen et al. 2014](#)). For this reason, we use the  $Z = 0.017$  tracks for comparison.

**Table 3.** Comparison of stellar properties between YREC, MIST, and PARSEC for  $1M_{\odot}$  at TAMS.

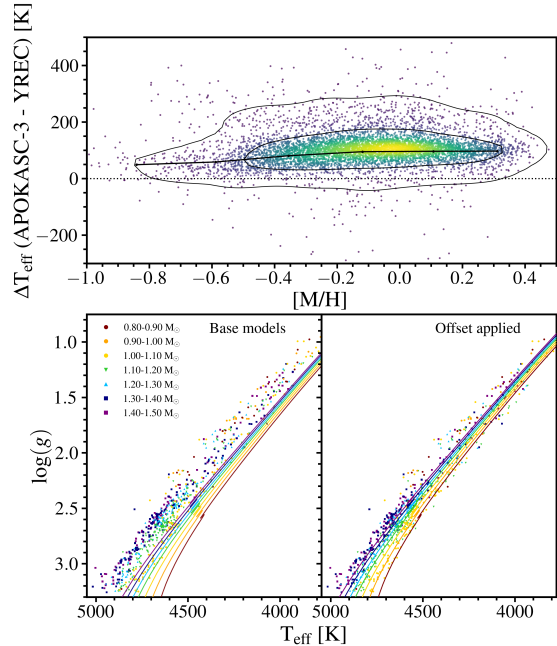
Parameter	YREC	MIST	PARSEC
Age [Gyr]	9.807	8.781	9.081
$\log(L/L_{\odot})$	0.235	0.260	0.223
$\log(T_{\text{eff}})$ [K]	3.763	3.766	3.765
$\log(R/R_{\odot})$	0.114	0.120	0.106
$\log(\rho_c)$ [ $\text{g cm}^{-3}$ ]	2.812	2.798	2.784
$\log(T_c)$ [K]	7.285	7.294	7.290

1215 surface boundary conditions, whereas this grid adopts  
1216 the Eddington-grey approximation.



**Figure 7.** H-R diagram showing evolutionary tracks for masses  $0.3$ ,  $1.0$ ,  $3.0$ , and  $8.0 M_{\odot}$  from the base grid. We show how each track evolves across metallicities  $[\text{Fe}/\text{H}] = -1.0$  (solid blue),  $-0.5$  (dashed green),  $0.0$  (dash-dotted purple), and  $+0.5$  (dotted grey). The  $0.3 M_{\odot}$  tracks extend to ZAMS, the  $1.0 M_{\odot}$  tracks to the tip of the RGB, and the  $3.0$  and  $8.0 M_{\odot}$  tracks to ZAHB, defined by  $Y_{\text{cen}} = 1 - Z - 0.06$ .

1217 Our final stop before the core He-burning phase is the  
1218 red giant phase, where we now have rich and precise  
1219 data sets due to asteroseismology. Stellar models make  
1220 strong predictions about the metallicity dependence of  
1221 stellar properties, as illustrated in Figure 7. We are now  
1222 in a position to test these predictions. Modern time do-  
1223 main surveys have revealed a rich landscape of stellar  
1224 variability. One of the highest impact results was the  
1225 discovery of solar-like oscillations in cool and evolved  
1226 giant stars. The global oscillation properties can be  
1227 combined with spectra to infer mass, radius and age  
1228 for large numbers of stars. In Figure 8, we compare our  
1229 models with data from the APOKASC-3 catalog (M. H.



**Figure 8.** *Top:* Difference between APOKASC-3 and YREC effective temperatures for the RGB sample in the APOKASC-3 catalog. Contours represent 68% and 95% of stars in the sample. *Bottom:* Kiel diagrams of APOKASC-3 RGB stars with metallicities in the range  $-0.05 < [\text{M}/\text{H}] < +0.05$ . The right panel shows the sample overlaid with the base grid solar metallicity tracks. The track colors correspond to the lower boundary of the mass bins. The left panel shows the same tracks after applying the temperature offset shown in the top panel.

1230 Pinsonneault et al. 2025). There is a clear offset, which  
1231 reflects the limitations of the solar calibration for stars  
1232 very different from the Sun. See J. Tayar et al. (2017)  
1233 for a discussion.

### 3.3.2. Core He-burning models

1235 The core He-burning phase is complex, and there can  
1236 be significant differences between evolutionary codes.  
1237 These complications arise in earnest even at helium ig-  
1238 nition. The onset of He burning is a smooth process in  
1239 high mass stars, though it does not proceed smoothly for  
1240 low-mass stars. For stars below  $\approx 2M_{\odot}$ , He-burning ig-  
1241 nites in a highly degenerate medium. Evolving a model  
1242 through this helium flash is numerically challenging be-  
1243 cause the usual stabilizing effects of the ideal gas equa-  
1244 tion of state are short-circuited. The assumptions of the  
1245 code that work well for most conditions - in particular,  
1246 instantaneous mixing of convective regions and exact hy-  
1247 drostatic equilibrium - become poor ones during the he-  
1248 lium flash. Great care with numerics is also required;

1249 the standard mesh allocation procedure in YREC works  
 1250 well for smooth and stable evolution, but less so for do-  
 1251 mains with rapid structural change. It is a testament to  
 1252 the strengths of the MESA code that it can routinely be  
 1253 evolved through the He flash (L. Bildsten et al. 2012). It  
 1254 is worth noting that YREC *can* solve these issues, and  
 1255 in fact it was one of the first codes to carry evolution  
 1256 through the helium flash (P. W. Cole et al. 1985). How-  
 1257 ever, this capability greatly complicated the code, and  
 1258 was removed from modern versions over many years of  
 1259 revisions.

1260 Instead, YREC uses a two-step procedure for low-  
 1261 mass horizontal branch stars that experience the he-  
 1262 lium flash. The equations of stellar structure can be  
 1263 solved in a time-independent fashion for stars with nu-  
 1264 clear energy sources. Given a composition profile, it  
 1265 is therefore possible for these stars to solve for a core  
 1266 He-burning model, similar to the approach used for the  
 1267 core H-burning main sequence. The most substantive  
 1268 difference is that the helium core mass, in addition to  
 1269 the total mass and metallicity, needs to be specified.  
 1270 For a self-consistent run, one evolves to the RGB tip,  
 1271 infers the core mass as a function of total mass and  
 1272 metallicity, and then rescales a seed core He burning  
 1273 model to the desired specifications. See Section 4.1 for  
 1274 a discussion of the procedure that we use. The current  
 1275 suite of seed models for core He-burning were built with  
 1276 the N. Grevesse & A. Noels (1993) mixture, and some  
 1277 namelists in the test suites also use the corresponding  
 1278 atomic opacity tables. It is not entirely trivial to adjust  
 1279 the non-uniform ZAHB abundances to a different mix;  
 1280 we intend to expand the suite to different mixtures in  
 1281 later releases. The default, however, is to use atomic  
 1282 opacity tables calculated with the GS98 mixture. Neu-  
 1283 trino cooling rates also differ from the default, taken  
 1284 from J. N. Bahcall (1989) in order to be consistent with  
 1285 the seed models. The helium flash models described in  
 1286 this section ( $< 2M_{\odot}$ ) use this two-step procedure.

1287 The precise physics appropriate for modeling the re-  
 1288 gions above the convective cores of the HB are not set-  
 1289 tled, which we discuss briefly here. The fundamental  
 1290 complication arises from carbon- and oxygen-rich ma-  
 1291 terial in the convective core ‘overshooting’ the nomi-  
 1292 nal convective boundary due to inertia from convec-  
 1293 tive motions at the boundary. Since this material is  
 1294 more opaque than the material it mixes into, this re-  
 1295 gion becomes unstable to convection according to the  
 1296 Schwarzschild criterion. Nevertheless, the region is sta-  
 1297 ble against convection when accounting for the compo-  
 1298 sition gradient in the Ledoux stability criterion.

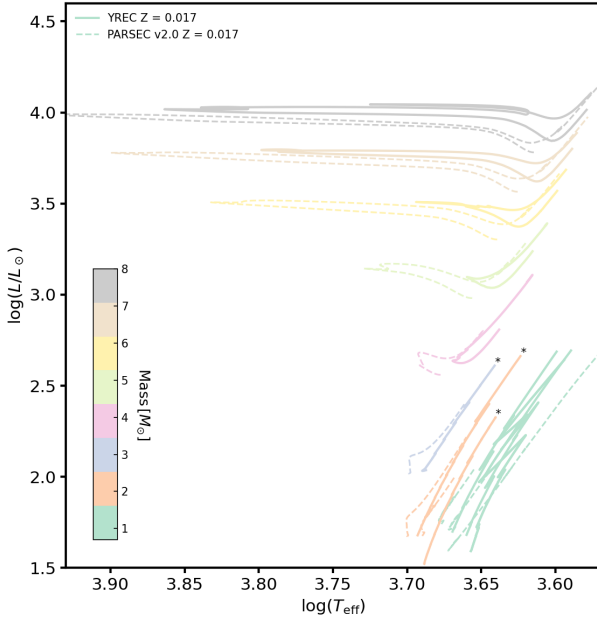
1299 It was therefore first proposed by P. Ledoux (1947)  
 1300 that such a ‘semi-convection’ region would experience

1301 strong enough convection to mix abundances, but not  
 1302 strong enough to significantly transport energy; i.e.,  
 1303 the region would transport energy via radiation. Dis-  
 1304 cussed in detail in the context of very massive stars (M.  
 1305 Schwarzschild & R. Härm 1958) and later in core helium  
 1306 core burning stars (e.g., B. Paczyński 1970), the typi-  
 1307 cal assumption is that the semi-convection region settles  
 1308 into a composition at every point such that  $\nabla_{\text{rad}} = \nabla_{\text{ad}}$ .

1309 An alternate formulation to the mixing dilemma above  
 1310 a core helium-burning model is overshooting, in which  
 1311 the convective boundary is only extended to a certain  
 1312 extent that depends on the motion of the convective el-  
 1313 ements as they pass the nominal convective boundary.  
 1314 Both prescriptions predict that the lifetime of the hori-  
 1315 zontal branch will be extended as the core grows, since  
 1316 it has access to more helium than it otherwise would.  
 1317 This results in a different ratio in the number of hori-  
 1318 zontal branch stars to asymptotic giant branch stars  
 1319 in clusters compared to a case when new helium is not  
 1320 introduced into the core (e.g. F. Caputo et al. 1989;  
 1321 T. Constantino et al. 2016). In detail, the evolution of  
 1322 the helium abundance proceeds differently under these  
 1323 two scenarios, which produces different predictions for  
 1324 asteroseismic gravity mode frequencies. Unfortunately,  
 1325 neither standard overshooting nor semi-convection re-  
 1326 produce satisfactorily, though non-standard models can  
 1327 (T. Constantino et al. 2015; D. Bossini et al. 2015, 2017).

1328 YREC adopts an overshooting formalism to model the  
 1329 physics of mixing around the core helium-burning core.  
 1330 We compare our horizontal branch models against PAR-  
 1331 SEC v2.0, since it, too, adopts an overshoot formalism  
 1332 (A. G. Bressan et al. 1981) that is different in detail, but  
 1333 similar in extent.

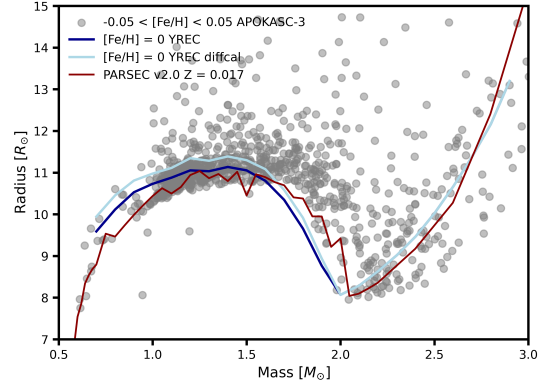
1334 We illustrate the evolution of our solar metallicity hori-  
 1335 zontal branch models from the ZAHB (defined to be  
 1336 when  $Y_{\text{cen}} < 1 - Z - 0.06$ ) in Figure 9, with tracks  
 1337 also shown from rotating PARSEC v2.0 (G. Costa et al.  
 1338 2019b,a; C. T. Nguyen et al. 2022b, 2025a) models,  
 1339 which cover the low-mass ZAHB via a similar core mass  
 1340 rescaling as YREC uses. The stepwise evolution in the  
 1341 core of low-mass stars is evident in YREC models, which  
 1342 reflects episodes where fresh fuel is ingested into the  
 1343 He-burning core. These are less evident in the PAR-  
 1344 SEC v2.0 models at the scale of the figure, although the  
 1345 qualitative behavior is present, as we show below when  
 1346 looking at the central helium evolution. Modest differ-  
 1347 ences in luminosity ( $\approx 10\%$ ) and in temperature ( $\approx 3\%$ )  
 1348 between low-mass YREC and PARSEC v2.0 ZAHB lo-  
 1349 cations in luminosity–effective temperature space are  
 1350 consistent with previous findings of agreement among  
 1351 publicly-available solar metallicity ZAHB models (D. An  
 1352 et al. 2019).



**Figure 9.** Comparison of the low-mass core helium burning phase from YREC (solid) and PARSEC v2.0 (dashed) from beginning of central core helium burning to  $Y_{\text{cen}} = 10^{-3}$  except where noted with \*. The tracks include masses between and including  $0.8M_{\odot}$  and  $2.4M_{\odot}$  in steps of  $0.4M_{\odot}$  and  $3M_{\odot}$  and  $8M_{\odot}$  in steps of  $1M_{\odot}$ . The low-mass PARSEC v2.0 models, which have suppressed helium ingestion episodes, generally reach core helium exhaustion at lower luminosities than YREC models, which experience helium ingestion episodes that allow for continued growth of the core and extension to larger luminosities.

1353 YREC models proceed to much higher luminosities  
 1354 and lower temperatures than PARSEC. This can be  
 1355 traced to a higher ingestion rate of helium into the core  
 1356 during overshoot in YREC, which lengthens the life-  
 1357 time of the horizontal branch. Importantly, PARSEC  
 1358 does not instantaneously mix abundances in convective  
 1359 regions, instead following a diffusion approach (C. T.  
 1360 Nguyen et al. 2022b). This latter difference would re-  
 1361 duce the amount of helium that enters into the core at  
 1362 any given time, and correspondingly reduce the sever-  
 1363 ity of mixing zone mergers compared to those in YREC,  
 1364 which assumes instantaneous mixing. (We discuss these  
 1365 mixing episodes below, which are also responsible for  
 1366 the sharp decreases in luminosity in Figure 9.)

1367 Asteroseismic data is a powerful test of stellar mod-  
 1368 els, and we compare ZAHB models with data in Fig-  
 1369 ure 10. We note that we define the ZAHB as when  
 1370  $Y_{\text{cen}} = 1 - Z - 0.06$  for YREC, which corrects for  
 1371 the energy required to lift degeneracy in the core. A  
 1372 sharp boundary in radius as a function of mass is ob-  
 1373 served in the data, which corresponds to the ZAHB.  
 1374 The handful of stars with radii below the edge are



**Figure 10.** Comparison of the core helium burning phase from YREC and PARSEC v2.0 against APOKASC-3 masses and radii for solar-metallicity stars. Cases with and without solar diffusion-calibrated  $Y = .273$  and  $Z = 0.0188$  are shown. YREC models with  $M \geq 2M_{\odot}$  use a two-step procedure. See text for details. Although there is qualitative agreement, asteroseismic constraints suggest opportunities for understanding the physics of the horizontal branch better in both YREC and PARSEC v2.0 models.

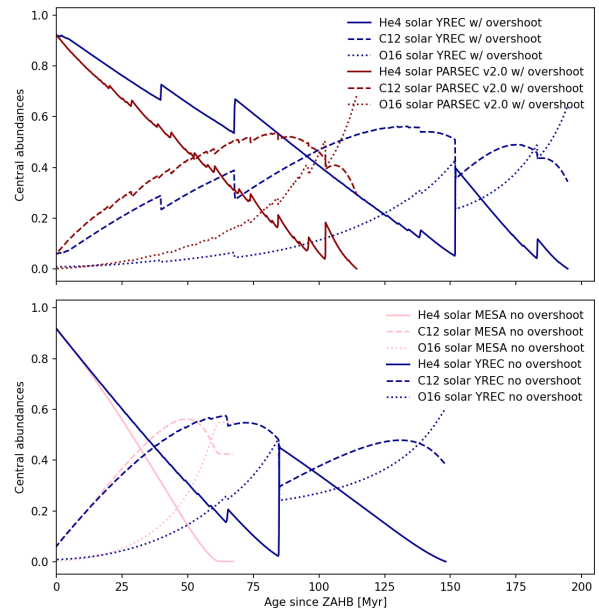
1375 the products of binary star interactions (Y. Li et al.  
 1376 2022). The predicted mass range for the onset of de-  
 1377 generate helium ignition agrees well with the local min-  
 1378 imum in radius at  $M \approx 2.0M_{\odot}$ . In detail, none of  
 1379 the models trace the lower boundary precisely. The  
 1380 core mass in the YREC case is constant to within 1%  
 1381 ( $M_{\text{core}} \approx 0.459M_{\odot}$ ) for  $0.7M_{\odot} \leq M \leq 1.2M_{\odot}$ . PAR-  
 1382 SEC v2.0 models have slightly higher core masses, sit-  
 1383 ting within 1% of  $0.475M_{\odot}$  until  $M \approx 1.35M_{\odot}$ . Above  
 1384  $M \approx 1.4M_{\odot}$  ( $M \approx 1.35M_{\odot}$ ), YREC (PARSEC v2.0)  
 1385 the core mass begins decreasing as neutrino cooling be-  
 1386 comes less efficient, which is reflected in the turnover  
 1387 of the mass-radius relation. Neither YREC nor PAR-  
 1388 SEC v2.0 completely agrees with the ZAHB location in  
 1389 mass-radius space. YREC models at higher masses may  
 1390 be brought into better agreement by adopting an ini-  
 1391 tial helium mass fraction consistent with the PARSEC  
 1392 choice (smaller than YREC’s by 0.015), while adopt-  
 1393 ing a higher helium abundance consistent with solar-  
 1394 calibrated diffusion models worsens the agreement (light  
 1395 blue in Fig. 10). Further work is clearly needed to recon-  
 1396 cile models with the powerful asteroseismic constraints.

1397 Turning to a closer inspection of the growth of the core  
 1398 with time, we show in Figure 11 the PARSEC v2.0 and  
 1399 YREC central abundance evolution of a solar horizontal  
 1400 branch model in the presence of overshooting. Initially  
 1401 helium is converted into carbon; at higher carbon abun-  
 1402 dance,  $C^{12} + \alpha$  becomes more efficient than triple  $\alpha$ ,  
 1403 leading to higher oxygen production.

1404 The small stepwise increases in helium visible in Fig-  
 1405 ure 11 and in the corresponding sharp decreases in lum-  
 1406 inosity in low-mass tracks shown in Figure 9 are due  
 1407 to the subtleties of the physics exterior to the convec-  
 1408 tive core mentioned above. The emergence of a semi-  
 1409 convection region will split the convective core into two  
 1410 if using the Schwarzschild stability criterion and adopt-  
 1411 ing the radiative temperature gradient in this region,  
 1412 as PARSEC v2.0 and YREC both do. The outer convec-  
 1413 tion zone shrinks in size even as the convective core  
 1414 grows with the opacity increase from nuclear burning.  
 1415 Overshooting helps bridge the gap at which point the  
 1416 core reaches the convective shell discontinuity, ingest-  
 1417 ing helium, shrinking due to the temporary decrease in  
 1418 opacity this causes, and splitting again. This split does  
 1419 not occur if the temperature gradient is assumed to be  
 1420 adiabatic and not radiative as is commonly assumed in  
 1421 semi-convection prescriptions, and these episodes do not  
 1422 occur. The lifetime of the horizontal branch is still in-  
 1423 creased in this case, though, as the core can mix with  
 1424 the semi-convection region. Classical breathing pulses  
 1425 as described in [A. V. Sweigart & P. Demarque 1973](#)  
 1426 and occurring at  $Y_{\text{cen}} < 0.1$  are also present in YREC  
 1427 models for both the overshooting and no overshooting  
 1428 cases. Such pulses are related to the smaller mixing zone  
 1429 merger episodes, though in this case the increased tem-  
 1430 perature of the core at this phase in the evolution can  
 1431 produce luminosities in the core large enough to over-  
 1432 come decreases in opacity due to helium ingestion; the  
 1433 result is a large increase in core size with helium inges-  
 1434 tion followed by quiescent increase in core size due to  
 1435 nuclear burning (e.g., [V. Castellani et al. 1971](#)).

1436 We compare against MESA r24.03.1 to further demon-  
 1437 strate differences in models, even in the absence of the  
 1438 details of overshooting. For this purpose, we modify  
 1439 the `1MS_pre_ms_to_wd` inlists from the MESA test suite,  
 1440 turning off mass loss as well as semi-convection and over-  
 1441 shooting, and adopting GN93 abundances with  $Z=0.02$ ,  
 1442  $Y=0.28$ . We also enforce the same mixing length for-  
 1443 malism as YREC ([E. Böhm-Vitense 1958](#)) and do not  
 1444 use Type 2 opacity tables in order to be consistent with  
 1445 YREC interior opacity tables. For this comparison, our  
 1446 YREC models use GN93 OPAL atomic opacities to be  
 1447 as consistent as possible with the YREC ZAHB seed  
 1448 models, which were initially generated using these abun-  
 1449 dances.

1450 As seen in Fig. 11, whereas for YREC we see small in-  
 1451 creases in central helium content as the convective core  
 1452 grows in size and ingests new helium, the same does not  
 1453 occur in MESA. The large jump at  $\approx 80$  Myr would cor-  
 1454 respond to the classical breathing pulses that occur at  
 1455 low helium content ( $Y < 0.1$ ). There is also a small dif-



**Figure 11.** Comparison of the core helium burning phase for a solar model from YREC (dark blue) with and without overshooting, matched to the core masses of PARSEC v2.0 (red; top) and MESA (pink; bottom). Note that MESA and PARSEC predict slightly different core masses:  $0.443M_{\odot}$  and  $0.469M_{\odot}$ , respectively, though most of the lifetime differences in the top and bottom panels is due to the difference in helium ingestion due to overshooting. A significant core breathing pulse is observed at  $\approx 80$  Myr in the no overshooting case. The evolution of the region outside the convective core in horizontal branch models even with similar treatments can differ.

1456 ference in the luminosity and the corresponding duration  
 1457 of the core He-burning phase from central  $\Delta Y = 0.06$   
 1458 until reaching  $Y_{\text{cen}} = 10^{-3}$ .

### 1459 3.4. Sample Cases

1460 It can be useful for readers to see examples of use  
 1461 cases for YREC, and we present some examples here.  
 1462 For solar models, and ones on the lower and upper MS,  
 1463 we discuss the relevant models from the test suite. We  
 1464 also include models with rotation and starspots, which  
 1465 include grids not given in the testsuite. The namelists  
 1466 for these cases can be found in the sample cases directory  
 1467 and are listed in Table 4.

#### 1468 3.4.1. Solar Models

1469 YREC has a long, and high-impact, history of usage  
 1470 for solar models starting in the 1960s ([P. R. Demarque  
 1471 & J. R. Percy 1964](#)). The first generation of solar models  
 1472 including rotation ([M. H. Pinsonneault et al. 1989](#)) was  
 1473 also achieved with YREC. The "standard" solar model  
 1474 for neutrinos in the 1990s was produced with YREC

**Table 4.** Families of sample cases discussed in the paper. The case prefix is presented in the first column. The location of the namelists (test suite or sample cases) is in the second, and a description is in the third. “Both” for the location is for situations where a subset of the cases are in the test suite.

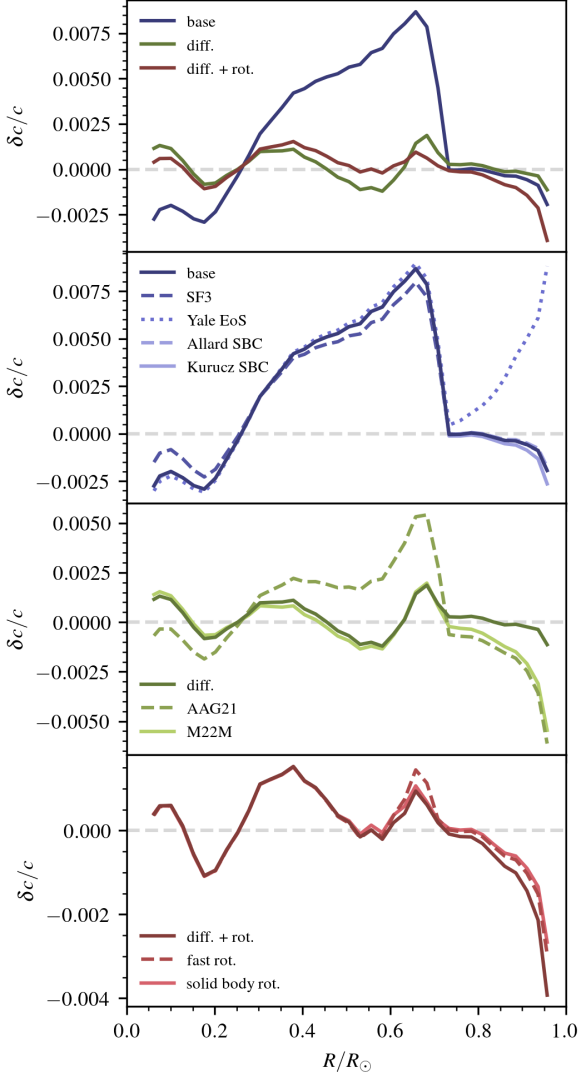
Case Name	Location	Descriptions
Test solar	Test Suite	Solar models calibrated with a range of input physics and birth mixtures
Sample base ZAMS	Sample Cases/LowerMS	Models of $0.09 - 1M_{\odot}$ evolved from the Dbl to the ZAMS.
Sample allard ZAMS	Sample Cases/LowerMS	Models of $0.03 - 1M_{\odot}$ , evolved from the Dbl to 15 Gyr ( $0.03 - 0.08M_{\odot}$ ) or the ZAMS ( $0.09 - 1M_{\odot}$ ) with Allard atmosphere SBCs
Sample kurucz ZAMS	Sample Cases/LowerMS	Models of $0.15 - 1M_{\odot}$ , evolved from the Dbl to the ZAMS with Kurucz atmosphere SBCs
Sample yaleeos ZAMS	Sample Cases/LowerMS	Models of $0.3 - 1M_{\odot}$ , evolved from the Dbl to the ZAMS with the Yale EoS
Sample m9p0feh0+0GS98 Cases	Both	$9M_{\odot}$ models with different EoS, degree of overshoot, with and without semi-convection
Sample Gyrochronology	Sample Cases	Models of $1.4M_{\odot}$ and below, including rotation, evolved from the Dbl to an age of 4 Gyr with different assumptions about angular momentum loss and internal angular momentum transport.
Sample Starspots	Sample Cases	Models of $1.4M_{\odot}$ and below, including star spots, evolved from the Dbl to the ZAMS with different star spot filling factors.
Sample ZAHB	Sample Cases/TAHB	Models of $1.8M_{\odot}$ and below, evolved from the ZAHB with core sizes consistent with the base case model grid of §3.3, with overshooting.
ZAHB diffcal	Sample Cases/TAHB	Same as ZAHB, but with diffusion-calibrated $Y$ and $Z$ .
Solar PARSEC	Sample Cases/TAHB	Solar model horizontal branch evolution matched to PARSEC v2.0, with overshooting.
Solar GN93 MESA	Sample Cases/TAHB	Solar model horizontal branch evolution matched to MESA, with GN93 opacity tables.

(J. N. Bahcall & M. H. Pinsonneault 1992; J. N. Bahcall et al. 1995), and these papers added gravitational settling as a key ingredient for solar models. YREC models also established the tension between a low solar metallicity and helioseismic data J. N. Bahcall et al. (2005); F. Delahaye & M. H. Pinsonneault (2006); F. L. Villante et al. (2014) YREC solar model work continues to the present day C. Basinger et al. (2024), including work on the faint young Sun problem. Our test suite includes a number of different solar models, calibrated under commonly used physical scenarios. Our sample cases account for differences in the mixture of heavy elements, the equation of state, nuclear reaction rates, surface boundary conditions, microscopic diffusion, the treatment of rotation, the initial angular momentum, and the associated mixing.

These are precise models, which require tight numerical controls that are not required for general purpose models. For example, the convergence criteria for these models can be difficult to achieve in the general case. Small timesteps in the pre-MS are required for precise measurements of lithium depletion, but not for a precise pre-MS lifetime or HR diagram evolution. For our solar models, this is achieved by lowering the maximum

change in the structure variables across a timestep relative to the general case. The inner fitting point is also moved inwards, for more precise solar neutrino calculations. This change has a negligible impact on broader classes of stellar models.

We show the sound speed profiles of our calibrated solar models in Figure 12. These results, as well as model solar neutrino fluxes, are in excellent agreement with literature values. See C. Basinger et al. (2024) for a detailed discussion. The solar models without microscopic diffusion show large deviations from the true solar profile, a well-known literature result (J. N. Bahcall & M. H. Pinsonneault 1992; J. Christensen-Dalsgaard et al. 1993). The choice of nuclear reaction rates and SBCs has a small impact, but the older Yale EoS is a worse fit to the data in the envelope. The birth mixture has a significant impact on the sound speed profile, although it is less than that for diffusion. The most metal poor mixture, AAG21, has the largest sound speed deviations. Rotationally induced mixing has a measurable effect, but broadly follows the same patterns as solar models with diffusion alone. Different initial angular momenta and models of internal angular momentum transport yield similar results.

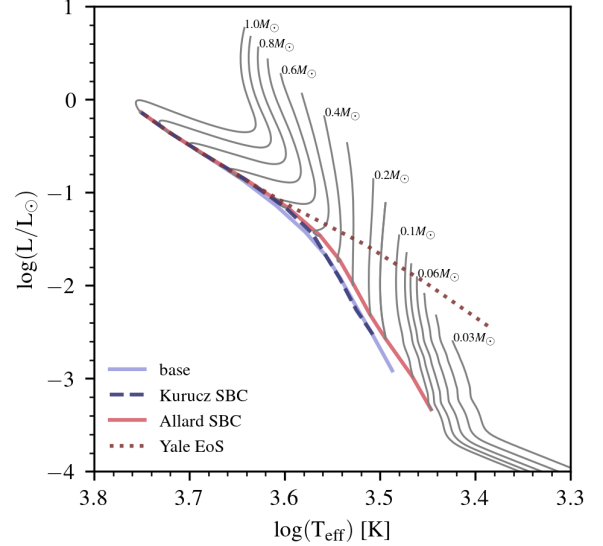


**Figure 12.** Fractional differences in sound speed between our test suite solar models and solar data from S. Basu et al. (2009). The top panel shows differences between models with the same mixture that include different physical processes. Differences in nuclear reaction rates and surface boundary conditions are compared in the second panel. The third panel shows the impact of changes in the heavy element mixture; all of these models included diffusion but not rotation. The impact of different initial rotation rates and the transport model are compared in the bottom panel.

### 3.4.2. The Lower MS

The choices for the surface boundary condition and the inclusion of star spots are quite important for lower MS stars, and we show the impact on the ZAMS for representative choices in Figure 13. The Kurucz and Allard table edges define the cool end of the runs. The general trend is that the more simplified models (Yale EoS and gray atmospheres) depart significantly from those with

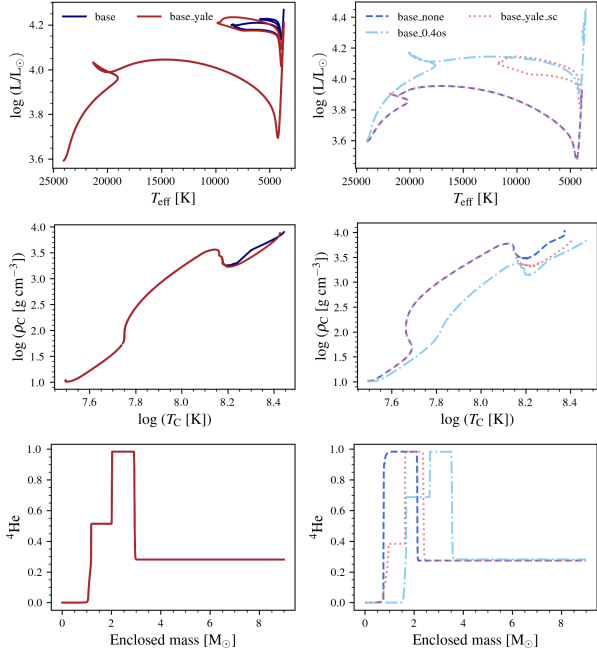
more sophisticated SBCs (Kurucz and Allard) and equations of state (OPAL and SCV) below about 4000 K. We only show the Yale EoS down to 0.3 solar masses, and the Kurucz atmospheres down to 0.15. For  $0.5 M_{\odot}$  and above, the Yale EoS and gray atmosphere case performs quite well on the main sequence. Because this combination is much faster to compute than the full case, this is a good choice for educational purposes. We also show brown dwarf tracks here, along with the onset of the brown dwarf cooling curves.



**Figure 13.** The impact of the choice of surface boundary conditions and equation of state on lower MS solar metallicity models. ZAMS curves are shown for the base models, Allard model atmospheres, Kurucz model atmospheres, and the Yale EoS. Model tracks to 15 Gyr (solid black lines) are shown for the Allard case.

### 3.4.3. The Upper MS

Example high-mass models are spread across the test suite and sample cases. Test suite models include two  $9 M_{\odot}$  stars evolved to the TAHB, one with the base set of input physics (**base**) and the other with the base set of input physics but the Yale EoS (**base\_yale**). Sample case models, selected to showcase different types of extra convective mixing, include three  $9 M_{\odot}$  stars evolved with base case physics and either 1) no overshooting and no semi-convection (**base\_none**), 2) overshooting with  $0.4 H_p$  (**base\_0.4os**), and 3) semi-convection and the Yale EoS (**base\_yale\_sc**). All models are run with all four levels of iterations. High-mass models evolving beyond the main sequence are very sensitive to adopted physics and numerical choices. Presently, the central fitting point must be moved in during the initial phase



**Figure 14.** The evolution and final helium profile of  $9 M_{\odot}$  models run with different assumptions about convection and two different EoSs. The left-hand panels compare the two test suite models, which largely overlap, and the right-hand panels compare the sample case models. Blue models are for the default OPAL EoS and red models are for the default Yale EoS.

1582 Note the two-tiered helium abundance in the cores of  
 1583 most models. This behavior, though peculiar, is linked  
 1584 to extra convective mixing, as it is seen in all mod-  
 1585 els with semi-convection or overshooting and not in the  
 1586 model without them. There ends up being a partially  
 1587 helium-depleted layer around the core where helium-rich  
 1588 material from the base of the helium shell was mixed into  
 1589 the core and burned. This partial He-depletion in mod-  
 1590 els with overshooting or semi-convection only shows up  
 1591 at intermediate masses ( $4\text{--}5 M_{\odot}$ , depending on adopted  
 1592 physics) and above.

#### 3.4.4. Main Sequence Gyrochronology

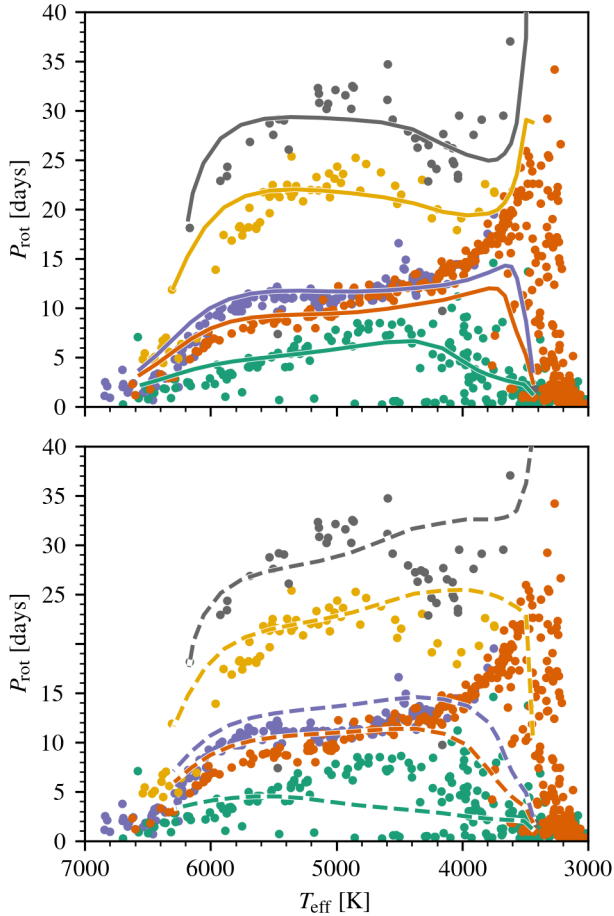
1593  
 1594 The use of open cluster rotation sequences to cali-  
 1595 brate both theoretical and empirical angular momentum  
 1596 loss models has a long history. Empirical fits to cluster  
 1597 sequences generally provide physics-agnostic interpolat-  
 1598 ing functions for period as a function of age and mass  
 1599 (e.g. E. E. Mamajek & L. A. Hillenbrand 2008; S. A.  
 1600 Barnes 2010; A. C. Lanzafame & F. Spada 2015; L. G.  
 1601 Bouma et al. 2023). Theoretical approaches leverage  
 1602 stellar structure models coupled to a physically moti-  
 1603 vated magnetic braking law, but remain fundamentally  
 1604 anchored to cluster (and other) calibrators (e.g. A. Kr-  
 1605 ishnamurthi et al. 1997; J. Bouvier et al. 1997; A. Sills  
 1606 & M. H. Pinsonneault 2000; P. A. Denissenkov et al.  
 1607 2010a; F. Gallet & J. Bouvier 2015; J. L. van Saders &  
 1608 M. H. Pinsonneault 2013; G. Somers & M. H. Pinson-  
 1609 neault 2016; L. Amard & S. P. Matt 2020; J. Roquette  
 1610 et al. 2021). We present an example of this calibration  
 1611 exercise with YREC rotating models in Figure 15. Two  
 1612 sets of physical assumptions are shown: one in which  
 1613 models include microscopic diffusion, internal angular  
 1614 momentum transport, and magnetized braking from the  
 1615 surface. A second suite shows the behavior of strongly  
 1616 magnetically coupled models, approximated by enforc-  
 1617 ing solid body rotation at all times. Both sets of models  
 1618 are independently calibrated by varying a normalization  
 1619 parameter  $f_k$  to match the rotational behavior at 0.9 so-  
 1620 lar masses in  $< 3$  Gyr old clusters (J. L. Curtis et al.  
 1621 2020).

1622 Models are run with mass-dependent initial rotation  
 1623 periods following G. Somers et al. (2017) and subject to  
 1624 saturated spin-down below Rossby numbers of  $0.1\text{Ro}_{\odot}$ .  
 1625 For those models allowed to differentially rotate, AM  
 1626 transport is performed using both hydrodynamical in-  
 1627 stabilities and an extra AM diffusion term following G.  
 1628 Somers & M. H. Pinsonneault (2016) that better repro-  
 1629 duces both the chemical and rotational histories of clus-  
 1630 ter stars. This diffusion term is constant in time, but  
 1631 a strongly varying function of stellar mass; we adopt  
 1632 the mass-dependence determined in G. Somers & M. H.

1557 of evolution for the later stages of evolution to run prop-  
 1558 erly. Models with semi-convection are particularly sensi-  
 1559 tive. Presently, a  $9 M_{\odot}$  model with semi-convection  
 1560 and the OPAL EoS does not reach the TAHB.

1561 Figure 14 shows the evolution in an HR diagram, the  
 1562 evolution of central density and temperature, and the  
 1563 final helium profile of the models. The test suite models,  
 1564 where only the EoS is changed between them, shown on  
 1565 the left-hand side of the figure, have nearly identical  
 1566 evolution, though the `base_yale` model lives  $\sim 1$  Myr  
 1567 longer. Central density and temperature diverge during  
 1568 core helium burning, corresponding to differences in the  
 1569 blue loops the models exhibit. Despite differences in  
 1570 core helium burning, the models end up with the same  
 1571 helium profile at the TAHB.

1572 Sample case models largely behave as expected. The  
 1573 `base_none` model evolves the fastest, does not exhibit  
 1574 a blue loop, and has the lowest luminosity and helium-  
 1575 core mass of the set. The `base_yale_sc` model evolves  
 1576 nearly identically to the `base_none` the model, deviat-  
 1577 ing during core helium-burning, when semi-convection  
 1578 becomes important in the region surrounding the core.  
 1579 The `base_0.4os` model has the highest luminosity and  
 1580 core mass, the longest lifetime, and the hottest and least  
 1581 dense core.



**Figure 15.** Rotating models compared against a select sample of open cluster rotation sequences from [J. L. Curtis et al. \(2020\)](#) and [D. Gruner et al. \(2023\)](#). Solid curves (top panel) denote models that include the full treatment of microscopic diffusion, internal AM transport, and wind braking. Dashed curves (bottom panel) represent strongly magnetically coupled models, where solid body rotation is enforced at all times.

[Pinsonneault \(2016\)](#) for display here. The braking is performed with the [J. L. van Saders & M. H. Pinsonneault \(2013\)](#) braking law.

Departures from the cluster sequences shown in [Figure 15](#) are generically areas of active research; the gyrochrones we show here represent a reasonable but not exhaustive exploration of the possible parameter choices and their optimization.

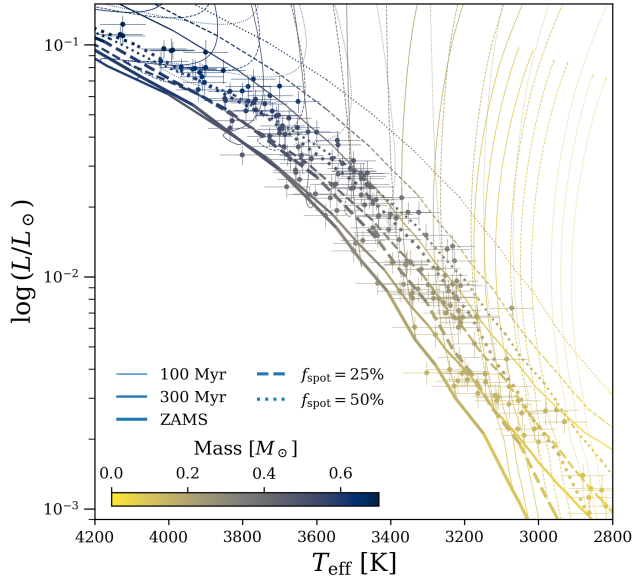
### 3.4.5. Young Stars and Star Spots

Starspots are frequently observed on the surfaces of cool, active stars. The presence of spots has long been associated with anomalies in color ([B. Campbell 1984](#); [J. R. Stauffer et al. 2003](#)), abundances ([D. R. Soderblom et al. 1993](#)), and spectra ([M. A. Gully-Santiago et al.](#)

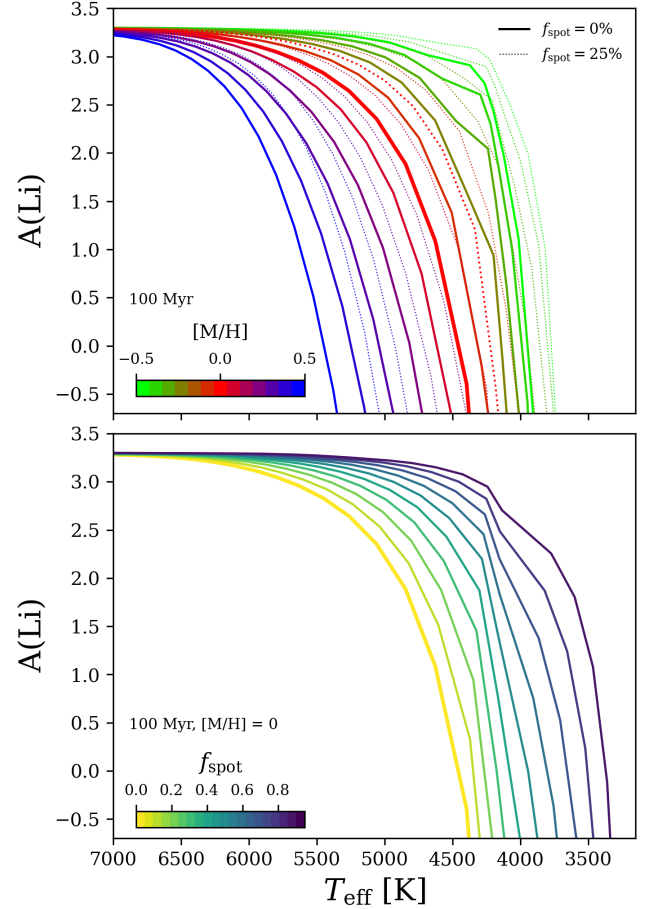
[2017](#)). As a magnetic activity proxy, it shows a clear relationship with the stellar Rossby number, similar to other activity proxies thought to have an origin in the stellar dynamo ([L. Cao & M. H. Pinsonneault 2022](#)). Spots have also associated with surface temperature systematics ([L. Cao & M. H. Pinsonneault 2022](#); [F. Pérez Paolino et al. 2025](#)), lithium depletion anomalies in young stars (e.g. [R. D. Jeffries et al. 2021, 2023](#); [L. Cao et al. 2026](#)), age systematics ([T. J. David et al. 2019](#); [L. Cao et al. 2022](#)), and radii excesses due to magnetic radius inflation ([R. J. Jackson et al. 2018](#); [L. Cao & K. G. Stassun 2025](#)). In YREC, starspots were initially implemented in [G. Somers & M. H. Pinsonneault \(2015\)](#) using the formalism from [H. C. Spruit & A. Weiss \(1986\)](#), and presented as a set of starspot isochrones in [G. Somers et al. \(2020\)](#). Here, we explore the present capabilities of YREC to study cool stars with starspots.

In [Figure 16](#), we show the [A. W. Mann et al. \(2015\)](#) sample of nearby field stars with empirically-calibrated masses and radii, with a set of solar-metallicity starspot isochrones overplotted at several representative ages (100 Myr, 300 Myr, and at ZAMS) and spot filling factors (0, 25%, and 50%). These models have gray atmospheres, are non-rotating, and have diffusion turned off, similar to the test suite case. The starspot isochrones trace the [A. W. Mann et al. \(2015\)](#) sample very well; though there is some metallicity variation which is not captured in this plot, many stars in the [A. W. Mann et al. \(2015\)](#) sample appear to depart from the solid lines, and their locations on this diagram are qualitatively better fit with some degree of magnetic inflation from starspots.

We subsequently run a series of runs observing the same microphysics conventions from the [G. Somers et al. \(2020\)](#) paper to produce [Figure 17](#): a set of lithium depletion patterns at 100 Myr, using the convention that  $A(\text{Li}) = \log_{10}(\text{Li7\_sur}) + 12$ . In this plot, we show non-rotating non-diffusive YREC models with a linear ramp from the Allard models at  $0.4 M_{\odot}$  to the Castelli–Kurucz models at  $0.6 M_{\odot}$ . In the top panel, we show the variation of the lithium depletion curve with respect to metallicity, with an unspotted and a spot-saturated model ( $f_{\text{spot}} = 25\%$  plotted). We see that the effect of spots is similar in some respects to a decrease in metallicity, though the difference in the shapes of the distributions is more complicated in detail. The bottom plot shows lithium depletion patterns for a solar metallicity case with different starspot filling factors, showing that the effect of spots is also roughly to mimic a depletion pattern at cooler temperature. This is expected, because spotted model stars have cooler convective zone bases as a consequence of magnetic inflation ([L. Cao et al. 2026](#)).



**Figure 16.** Starspot isochrones at solar composition evaluated at 100 Myr, 300 Myr, and the ZAMS (defined where  $X_{\text{cen}}/X_{\text{cen},0} = 0.993$ ); and 0 (solid), 25% (dashed) and 50% (dotted) spot filling factor. Mass tracks are shown for constituent tracks evolved to the ZAMS. The *A. W. Mann et al. (2015)* sample is overplotted with reported errors. Colors are assigned by the stellar mass, as shown in the colorbar.



**Figure 17.** Starspot lithium isochrones evaluated at 100 Myr for various metallicities and starspot filling factors. Top plot shows the variation of the lithium abundance pattern with metallicity (between  $-0.5$  and  $+0.5$ , stepping by  $0.1$  dex), for unspotted (solid) and 25% spotted (dotted) cases. Bottom plot shows the variation of a 100 Myr solar metallicity abundance pattern with starspot filling factor from  $f_{\text{spot}} = 0.0$  to  $0.9$ , stepping by  $0.1$ .

1699

### 3.4.6. Rotation Profiles in Evolved Stars

1700 While many main sequence stars appear to be rotat-  
 1701 ing as close to solid bodies by the end of the main se-  
 1702 quence, it is not clear that red giant branch stars with  
 1703 large envelopes and shorter lifetimes should do the same.  
 1704 In fact, the available asteroseismic results suggest that  
 1705 the cores of these stars are rotating significantly faster  
 1706 than their envelopes (*J. Tayar et al. 2019; G. Li et al.*  
 1707 *2024*). It is therefore useful to be able to parameter-  
 1708 ize the radial rotation profiles in these stars in order  
 1709 to explore various limiting cases. YREC offers the op-  
 1710 tion to force the entire star to rotate as a solid body or  
 1711 conversely, to conserve angular momentum within each  
 1712 zone (*J. Tayar & M. H. Pinsonneault 2013*). There are  
 1713 also options to enforce an angular momentum transport  
 1714 timescale, or allow only hydrodynamic mechanisms to  
 1715 act (*G. Somers & M. H. Pinsonneault 2016*). Finally,  
 1716 YREC also has the option to allow differential rotation  
 1717 in convection zones, where the differential rotation pro-  
 1718 file has the rotation frequency  $\omega$  scaling with the radial  
 1719 coordinate  $r$  to some power between  $-2.0$  (constant spe-  
 1720 cific angular momentum) and  $0.0$  (solid body rotation).  
 1721 When setting up the differentially rotating convection  
 1722 zone, the code also has the option to enforce solid body  
 1723 rotation below the surface convection zone, as well as  
 1724 the option to force the entire interior to rotate at the

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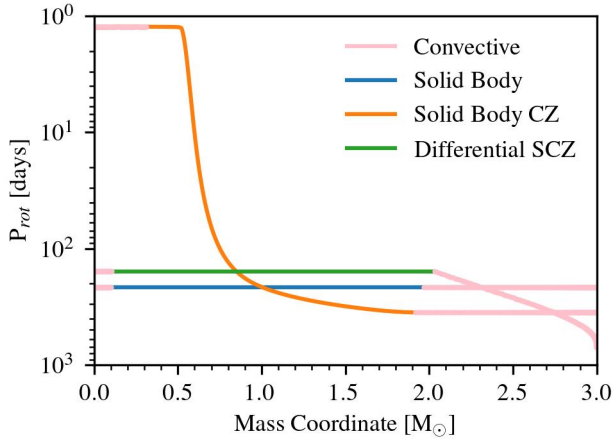
1735

rate of the base of the convection zone. We show ex-  
 amples of these various cases for a  $3 M_{\odot}$  star halfway  
 through core-helium burning in Figure 18.

## 4. SCRIPTS

The YREC code was designed to run single ini-  
 tial value problems to a specified endpoint. How-  
 ever, there are a number of cases where scripts  
 are essential or convenient. We provide tools and  
 scripts in the YREC Wrappers folder<sup>14</sup> of the YREC  
 GitHub repository. Within this repository are the  
`modelgrid_tools/main_tools` repository, where the

<sup>14</sup> [https://urldefense.com/v3/\\_https://github.com/yreclab/yrec/tree/main/wrappers/modelgrid\\_tools](https://urldefense.com/v3/_https://github.com/yreclab/yrec/tree/main/wrappers/modelgrid_tools)



**Figure 18.** Radial rotation profiles for a  $3 M_{\odot}$  star halfway through helium burning. Convective regions are pink in all cases. We enforce various rotation profiles including solid body for the whole star (blue), a solid body interior and a differentially rotation surface convection zone (green), and solid body convection zones but a differentially rotating radiative region (orange).

base tools for new users are found. In addition, we feature the `modelgrid_tools/alternate_tools` repository, which houses our more advanced tools for experienced users and large workflows. We recommend starting with the `main_tools` repository and using the `alternate_tools` for large workflows, parallel processing, and backups in case the file readers in the `main_tools` fail. All scripts and wrappers are written in Python unless otherwise noted. We provide links to and brief descriptions of these tools in Appendix A. The specific tools released with this paper include:

- A means of building the executable code for different compilers and common compile options.
- A namelist converter, to customize the output and input directories for the code.
- A YREC output file reader.
- Scripts for running grids of models with different initial mass, composition, or rotation rate.
- A script to calibrate solar models, using the results of prior runs to estimate best parameters for future ones.
- Checks on the internal consistency of the input physics tables, used prior to running the code.
- Checks on the numerical precision of the calculations.

- Scripts for running the code in parallel, on multiple processors.

There are three other families of tools that we discuss in more detail here. The first is our procedure for restarting models after the He flash, which is more involved than typical YREC use cases. The second is the linkage of YREC with the `kiuhoku` tool, in the context of building isochrones from user-supplied grids of stellar models. We also discuss the `rotevol` code, which is a fast and efficient tracer code for generating angular momentum evolution scenarios from grids of standard stellar models.

Our final sub-section contains some sample exercises, illustrating the power of YREC as a teaching tool.

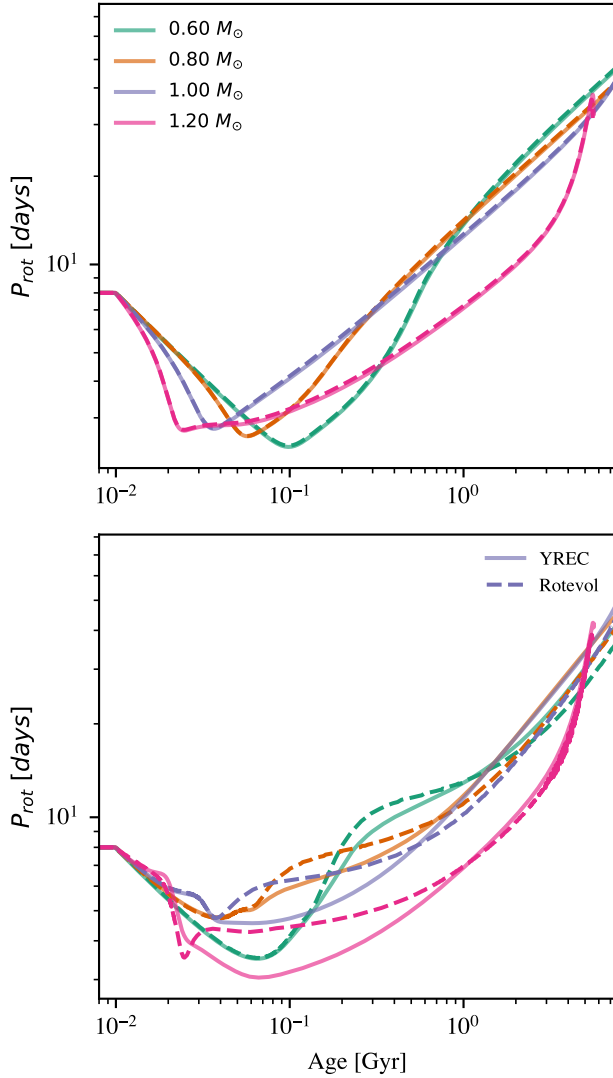
#### 4.1. RGB Tip to ZAHB

Users may generate custom zero-age horizontal branch models by re-scaling the envelope mass and the core mass of the YREC seed ZAHB models to match those of a user-generated tip-of-the-RGB model. In this way, a user may run a physically consistent calculation from the pre-MS to the ZAHB, bypassing the helium flash. An example of the envelope mass rescaling procedure is provided in the test suite (see Table 2). An example is also provided for the core mass rescaling procedure in the context of the solar mass evolution run listed in Table 2. This mass rescaling approach has been used in the latest PARSEC v2.0 models (C. T. Nguyen et al. 2025b), and may be important, depending on the use case, to preserve consistency of the ZAHB models with those from previous stages of evolution (e.g., for isochrone generation purposes).

#### 4.2. Tracer Angular Momentum Evolution Grids

For low mass stars there are a number of degrees of freedom in the model for angular momentum loss and in the initial conditions for rotation. The parameters in the model are usually constrained empirically, by matching the observed time evolution in star clusters against suites of theoretical models with multiple degrees of freedom.

For example, angular momentum loss is a steeply increasing function of rotation rate with a saturation threshold. Slow rotators are insensitive to this saturation threshold for the angular momentum loss law, but rapid rotator histories are strongly influenced by it. Core-envelope decoupling, by contrast, is a mass-dependent process whose primary observational signature is the spin down of slower rotators. At late stages, the asymptotic slope depends on the assumed dynamo scalings, while the zero point of the loss model sets the absolute value of the asymptotic trend. It can therefore



**Figure 19.** A comparison between full YREC rotation models and `rotevol` forward models for tracks with masses 0.6, 0.8, 1.0, and 1.2  $M_{\odot}$ . The upper panel shows the comparison between the predicted rotation periods under the solid body approximation. The lower panel shows the difference in predicted rotation periods for non-solid body rotation with core-envelope coupling timescales given by P. A. Denissenkov et al. (2010b).

1821 best-fit solution relative to a user-defined figure of merit.  
 1822 `rotevol` has already seen extensive use in the literature  
 1823 (J. L. van Saders & M. H. Pinsonneault 2013; J. L. van  
 1824 Saders et al. 2016, 2019; G. Somers et al. 2017; T. S.  
 1825 Metcalfe et al. 2020; N. Saunders et al. 2024; F. Chiti  
 1826 et al. 2024; Y. Li et al. 2025; L. Cao et al. 2023; L. Cao  
 1827 & K. G. Stassun 2025); this represents the first public  
 1828 release of these scripts.

1829 In addition to the test suites, we have included scripts  
 1830 which allow the user to run `rotevol` on pre-computed  
 1831 model grids. These scripts include functionalities for  
 1832 generating rotation tracks for different physical paramet-  
 1833 ers and running a grid of models through `rotevol`. As  
 1834 `rotevol` forward models the rotational evolution of a  
 1835 pre-computed stellar track, one of its main advantages  
 1836 is speed. Compared to a full YREC run with rotation,  
 1837 `rotevol` reduces the computation time by 2 orders of  
 1838 magnitude. `rotevol` takes on order a tenth of a second  
 1839 to compute the rotation evolution of a mass track.

1840 Using `rotevol`, we have computed comparison cases  
 1841 between rotating YREC models and `rotevol` forward  
 1842 models for stellar masses between  $0.2M_{\odot}$  and  $1.2M_{\odot}$ .  
 1843 Figure 19 compares the predicted rotation periods from  
 1844 YREC and `rotevol` for the two rotation scenarios dis-  
 1845 cussed in Section 3.4.4: solid body rotation and core-  
 1846 envelope decoupling. The figure shows `rotevol` runs  
 1847 with identical braking law parameters on tracks com-  
 1848 puted with identical physics to those in Section 3.4.4,  
 1849 but with rotation off. The solid body case produces  
 1850 excellent agreement (within  $\approx 5\%$ ) in the predicted ro-  
 1851 tation periods compared to fully rotating YREC mod-  
 1852 els. The `rotevol` models with core-envelope decou-  
 1853 pling show disagreements up to  $\approx 20\%$  for stellar masses  
 1854  $\geq 1.1 M_{\odot}$ —discrepancies that are not unexpected, given  
 1855 that the underlying physical treatment of the internal  
 1856 angular momentum transport is different (see also P. A.  
 1857 Denissenkov et al. 2010a; G. Somers & M. H. Pinson-  
 1858 neault 2016; F. Spada & A. C. Lanzafame 2020).

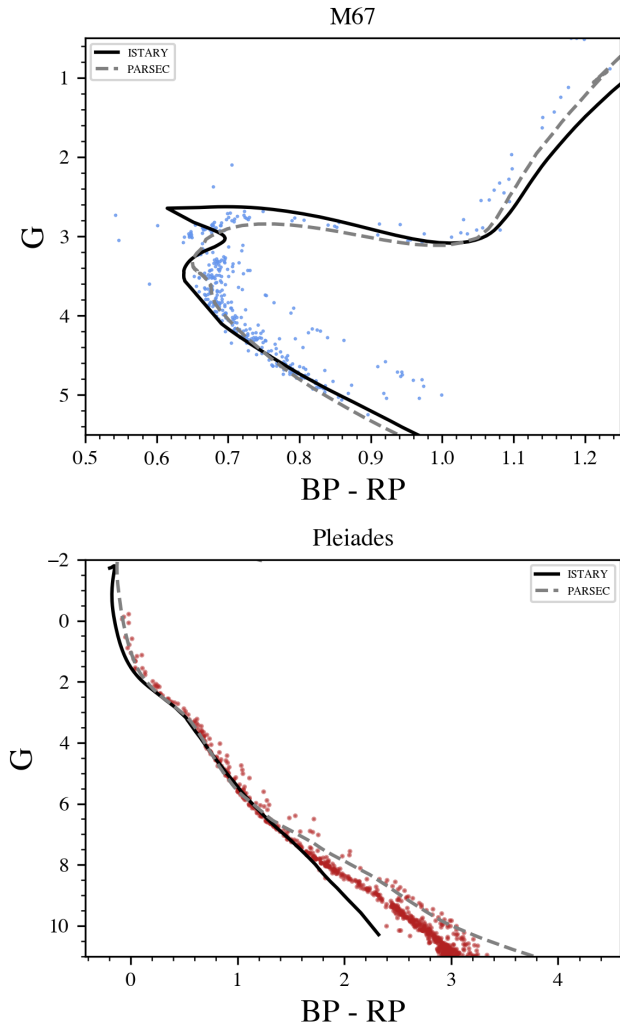
1859 Users can find the `rotevol` code in the examples folder  
 1860 of the YREC release code. This folder includes scripts  
 1861 for running `rotevol` in non-rotating YREC tracks found  
 1862 in the sample output folder.

### 1863 4.3. *ISTARY* - The YREC Stellar Isochrone Set

1864 Isochrones are elegant tools for studying stellar pop-  
 1865 ulations. Here we present a sample set derived from  
 1866 our grid, a guide for isochrone creation in the form of a  
 1867 Jupyter .ipynb file, and a visualization of the results.  
 1868 The Isochrones and Stellar Tracks for Astrophysics  
 1869 Research by the YREC Collaboration (ISTARY)<sup>15</sup>

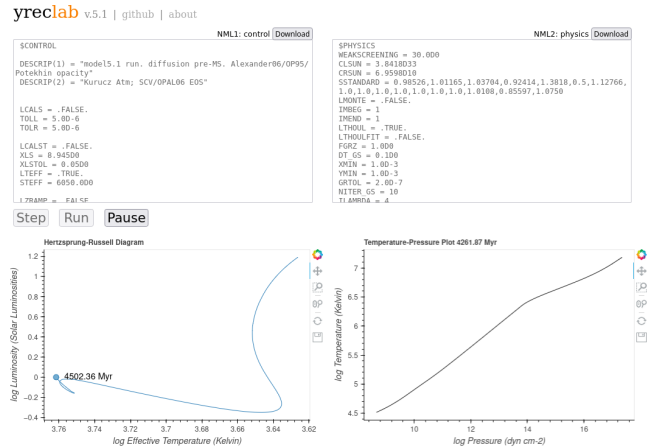
<sup>15</sup> [https://github.com/avincesmedile/yrec\\_isochrones](https://github.com/avincesmedile/yrec_isochrones)

1811 be useful to quickly generate large suites of models, with  
 1812 the intention of obtaining a best fit solution in a higher  
 1813 dimensional space. For slowly rotating stars the struc-  
 1814 tural effects of rotation can be usefully treated as mod-  
 1815 est perturbations to the structure. We have therefore  
 1816 developed a program, `rotevol`, which reads in model  
 1817 tracks generated without rotation, and generates what-  
 1818 if scenarios for surface rotation as a function of time.  
 1819 `rotevol`, in turn, can be called by scripts that perturb  
 1820 the assumptions in the physical model and search for a



**Figure 20.** A CMD example of the isochrone set generated from the YREC sample grid. We plot the absolute Gaia DR3 G mag and Gaia DR3 BP-RP. Isochrones (black) were computed using the `kiauhoku`-generated EEPs derived from the YREC main grid. We adopted solar metallicity for M67 and an age of 3.95 Gyr (C. Reyes et al. 2024), and  $\text{Fe}/\text{H} = 0.03$  with an age of 0.125 Myr (S. Gossage et al. 2018) for the Pleiades. Data for M67 and the Pleiades were taken from C. Reyes et al. (2024); M. G. J. van Groeningen et al. (2023), respectively. Isochrones are compared with Pleiades data (bottom panel) and M67 data (top panel).

1870 isochrone set was produced using our custom isochrone  
 1871 functions in conjunction with the `kiauhoku` stellar inter-  
 1872 polator code from Z. R. Clayton et al. (2020a). `kiauhoku`  
 1873 takes a grid of YREC .track files and converts it into a  
 1874 multi-index pandas dataframe by common initial mass,  
 1875 mixing length, model number, and metallicity. Then,  
 1876 it converts this dataframe into equivalent evolutionary  
 1877 phases (EEP) using the method by A. Dotter (2016).  
 1878 We then create isochrones using a custom wrapper,



**Figure 21.** The `yreclab` web interface, showing a run in progress, with toggles to “step” (evaluate one timestep), “run” (evaluate until paused or execution ends), and “pause” the execution.

1879 `make_iso()`. ISTAR and our isochrone creation guide  
 1880 can be found in the YREC Github repository.

1881 Our sample isochrones are for  $[\text{Fe}/\text{H}] = 0.0$  and cover  
 1882  $0.250 \leq \text{Gyr} \leq 13$ , although others can be generated  
 1883 with the same machinery. We compare with data in  
 1884 the Pleiades and M67 open clusters, which we show in  
 1885 Figure 20. The isochrones here were converted into bolo-  
 1886 metric magnitudes using the YBC Code Y. Chen et al.  
 1887 (2019). For the Pleiades, we employ individual distances  
 1888 for members derived from Gaia DR3 parallaxes, and uti-  
 1889 lize an extinction value of  $A_v = 0.1$ . We find good agree-  
 1890 ment between ISTAR, PARSEC, and the data up to  
 1891 the lower MS, where known discrepancies between the  
 1892 Pleiades and isochrones occur Y. Chen et al. (2019). For  
 1893 M67, we used the distances (an average distance modu-  
 1894 lus of 9.614) and extinctions from C. Reyes et al. (2024).  
 1895 We find good agreement between ISTAR, PARSEC,  
 1896 and the M67 data.

#### 4.4. YREC as a Tool for Stellar Structure and Evolution Classes

1897  
 1898  
 1899 Stellar structure and evolution is a core topic for  
 1900 undergraduate and graduate education in astronomy.  
 1901 The two topics—structure and evolution—demand dif-  
 1902 ferent approaches. Stellar structure is deeply tied to  
 1903 the physics of stars, and most instructors adopt a semi-  
 1904 analytic approach close in nature to the tactics used in  
 1905 core physics courses. Stellar evolution, however, is more  
 1906 typically treated as a distillation of results. YREC is an  
 1907 excellent tool for both structure and evolution. Students  
 1908 can compare the idealized results from analytic structure  
 1909 calculations to those from full models, and they can gen-  
 1910 erate their own evolutionary histories. We have provided  
 1911 some exercises in the examples folder. The first exercise

compares pressure as a function of density, temperature and composition to that predicted by analytic expressions used in classes, such as those for radiation pressure or the ideal gas law. The second exercise illustrates the effects of significant variations in entire families of rates (CNO and pp) as opposed to selective changes in single rates (pp and  $N^{14} + p$ ). The third has students compare analytic estimates for the equilibrium abundances of elements that participate in CNO burning to their model abundances.

Another unique teaching application of YREC is the web laboratory, also known as `yreclab`. Due to the efficient design of the code and its lightweight nature, it has been possible to compile YREC to WebAssembly with the code `emscripten` (A. Zakai 2011), which allows it to be run interactively client-side as a modern single-page web application hosted on Github Pages<sup>16</sup> (L. Cao & M. H. Pinsonneault 2026). An example run is shown in Figure 21. `yreclab` fully supports the current YREC implementation and its namelists can be modified just like the desktop version of YREC. Modifications to the code<sup>17</sup> allow it to be run in an interpreted state, and the stellar evolution code can be directly controlled via the Javascript scripting language—allowing it to be paused and restarted to examine the models of a run in progress. During any point of the evolutionary run, the virtual file system allows all files created during the run to be downloaded.

These architectural choices make `yreclab` uniquely suited for teaching & experimentation in stellar astrophysics (L. Cao & M. H. Pinsonneault 2026). Existing efforts such as `MESA-Web` (C. E. Fields et al. 2023) have been successful at providing a platform for teaching stellar physics, but require calculations to be submitted for later download by email, and have a limited subset of toggles available on their calculation submission page. With `yreclab`, the scriptable WebAssembly platform allows calculations to be supervised and interacted with at real-time, at near native desktop speeds (within a factor of two), with full support for the same namelists available in the desktop version of YREC.

## 5. SUMMARY

Stellar astrophysics is a rich topic, and it has deep connections across astronomy writ large. Stellar evolution codes are natural tools for understanding the life cycles of stars, which is why a number of were developed over the years. However, the list of public codes is short, and different codes are optimized for different kinds of stars.

In this paper we present a public release of the YREC stellar evolution code. YREC has been primarily used for the evolution of low and intermediate mass stars, and in that context it is a state-of-the-art code with sophisticated options. It can be run for a wide range of conditions, but caution should be taken when using any tools outside of the domains where development has been focused. We begin by summarizing the key domains for use, followed by the road map for improvements and updates. For the discussion below, we distinguish between brown dwarfs ( $M < \sim 85M_J$ ); low mass stars with deeper convective envelopes ( $\sim 1.3M_\odot > M > \sim 85M_J$ ), intermediate mass stars ( $\sim 8M_\odot > M > \sim 1.3M_\odot$ ), and high mass stars ( $M > \sim 8M_\odot$ ), with these ranges defined at solar metallicity.

YREC has been extensively used for low mass stars, starting from the pre-MS through core He burning. It can be used for hotter brown dwarfs, but more realistic models are limited by the surface boundary condition and equation of state tables to either younger ages or higher mass. In its current form we recommend caution below  $30M_J$ .

The code requires a starting model, but fortunately rescaling allows users to generate starting models with mass and composition different from those in the library. There is also no rule against exporting models from other codes to YREC. Our initial conditions, on the upper Hayashi track, begin in a fictional place. Fortunately, it's also a useful one, as these models can be rescaled and evolved to a more physically reasonable deuterium birthline. Even in this case, real models would accrete up the birthline, so the real very early evolution could be quite different than that in the models. This has limited impact on classical model properties, but it's important for evolving models with rotation. Artificially large starting models will have extravagant angular momenta, so care needs to be taken to choose a proper point for initializing such models.

The code allows a range of angular momentum evolution models, including the structural effects of star spots, which will be especially useful for gyrochronology and rotationally induced mixing. Strong magnetic coupling, such as that inferred from the Taylor-Spruit instability, can be effectively modeled as solid body rotation for these purposes. Wave-driven angular momentum transport is not currently supported in the code. YREC makes state of the art solar models.

The code also treats microscopic diffusion well in the low mass star domain. The primary limitation is that metals are diffused at the same rate. We note, however, that this is of order a 1% level effect for solar models. The code crashes at the He flash, but our scripting tools

<sup>16</sup> <https://yreclab.github.io/>

<sup>17</sup> <https://github.com/yreclab/yreclab>

can be used to restart evolution in the core He-burning phase. The current code does not include support for the thermally pulsing AGB phase, nor does it include thermohaline mixing. The code does include mass loss, but it is being re-worked and the current version is not supported.

For intermediate mass stars, numerical issues arise with microscopic diffusion in stars with thin surface convection zones. These can be avoided by setting the outer fitting point deep enough, at the cost of shutting the microscopic diffusion calculations off. The detailed behavior of the code in the outer layers will depend on the location of the outer fitting point, which should be chosen with care. Note that this has very little impact on ages, or standard isochrones, as radiative envelope stars are insensitive to the treatment of the outer boundary condition. YREC has limited options for the treatment of convective boundaries, which can become important in this mass domain. Users interested in rotation should ensure that the initial angular momenta of the models are reasonable for the stars under consideration.

The upper mass range of YREC is set by the onset of radiation-pressure driven winds. These induce negative pressure in the atmosphere solutions above 16 solar masses - a physical, but regrettable, feature in a system with logarithmic co-ordinates. YREC at present does not include radiation pressure driven winds, nor does it include nuclear burning stages beyond helium burning. It does capture the broad properties of models in the domain where it runs; above 12 solar masses, the code should be used with caution.

YREC will run with a wide range of birth mixtures, although the input physics tables in general do not support changes in the heavy element mixture. The maximum  $Z$  for the tables is 0.1, and users should be prepared to face challenges in the limit where the metal content is taken to zero. The isotopes and species that the code can track are hard-coded at present, although work is underway to generalize this.

The code continues to be actively developed. We intend to overhaul and bring the code up to date for the treatment of convective boundaries and mass loss. More generally, we are looking at adding in well-posed treatments of important physical processes. We are also in the process of evaluating and updating the basic input physics. This is an effort where we welcome external effort. Over a longer term, we intend to make the code easier to modify, and to generalize some of the hard-coded components that could make such changes more challenging. It is our hope that this code proves to be useful, and we welcome feedback.

## ACKNOWLEDGMENTS

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The YREC stellar evolution code has been the product of decades of effort and improvements from a variety of contributors. Many, but not all, of these contributions are preserved in comments in the code. Major contributors in addition to the authors on this paper include Pierre Demarque, Richard Larson, Michael Prather, Don van den Berg, David Guenther, Grant Newsham, Brian Chaboyer, and John Bahcall. Lawrence Pinsonneault also made major contributions to the organization and documentation of the code.

MHP acknowledges support from NASA grants 80NSSC24K0622 and 80NSSC24K0637. JT and LM acknowledge support from 80NSSC22K0812. JvS acknowledges support from NSF grants AST-2205888 and AST-2507789. ZRC acknowledges support from NASA ROSES 80NSSC24K0081. LC acknowledges support from the Vanderbilt Initiative in Data-intensive Astrophysics (VIDA) and NASA Grant 80NSSC24K0622.

## AUTHOR CONTRIBUTIONS

MP is responsible for a large fraction of the YREC stellar evolution code. He also took primary responsibility for writing the text and developing the test suite, as well as documenting the code. JvS contributed to code testing and development, gyrochronology example cases, and documentation. LC contributed to code development, documentation, starspots example cases, test suite validation, and the `yreclab` web interface. JT contributed to the development of the radial differential rotation modules, the available atmosphere boundary condition options, assisted with grid tool development, and participated in writing and documentation. FD contributed opacity tables, some writing of the code, and he participated in the writing of the manuscript. LM contributed to the grid tool development, generation of the base stellar model grid, and writing of the relevant sections of the manuscript. RAP tested the structural and evolutionary behavior of high-mass models with various modifications to convection and contributed to associated portions of the manuscript. VAS wrote the scripts for installing the base model grid into the `kiahoku` stellar model interpolator, the guide to creating isochrones with either a custom or the base YREC grid, and the `ISTARY` code/library. In addition, VAS organized the user tools repository, contributed to grid tool development, and wrote relevant sections of the manuscript. SB contributed to the grid tool development. AA contributed to the scripts for the angular momentum tracer

code and the relevant manuscript sections. KC participated in the maintenance and modernization of the YREC code and related discussions. MR developed the build system, test suite executor, hunted down bugs in the YREC code, and contributed to relevant sections of the manuscript. ZRC authored the `kiauhoku` package, advised on its usage, and provided feedback on

the manuscript. JCZ contributed to horizontal branch model testing and associated manuscript writing.

*Software:* `astropy` (Astropy Collaboration et al. 2013) `kiauhoku` (Z. R. Clayton et al. 2020a,b),

## APPENDIX

### A. SCRIPTS AND TOOLS

#### A.1. Building YREC

Users can download YREC from <https://github.com/yreclab/yrec>. YREC is implemented entirely in Fortran 77 and is straightforward to build. The code is Fortran 90 compliant. A simple build system is provided to create binaries of YREC for several different purposes. Two major compiler families are currently supported, the GNU Fortran compiler (`gfortran`) and the Intel Fortran compiler (`ifort`, `ifx`). Building the software is supported on Linux, Mac OS, and Windows. The default build configuration is set to generate a program with the best performance for large scale model runs. Development and debug build configurations are also provided which may be useful to those who wish to modify or further develop YREC.

#### A.2. Namelist Converters

The YREC control namelists (`nml`) contain file paths to input files containing physical parameters and input/output controls. Users can run examples using the relative paths provided, or edit the locations of input and output files manually or by utilizing the `change_nml()` or `update_nml` codes. We provide a simpler namelist tool for new users, `update_nml`<sup>18</sup>, which allows users to modify namelists one variable at a time using command-line prompts and inputs. In addition, we provide the more advanced `change_nml`<sup>19</sup> tool, which can intelligently modify multiple namelist files at once to specify input and output data locations for YREC. After modification, users can run their YREC namelists either from the command line or via the Python wrappers introduced here for coordinated parallel runs, with the outputs deposited in an organized fashion.

<sup>18</sup> [https://github.com/avincesmedile/YREC-Wrappers/blob/main/modelgrid\\_tools/main\\_tools/update\\_nml.py](https://github.com/avincesmedile/YREC-Wrappers/blob/main/modelgrid_tools/main_tools/update_nml.py)

<sup>19</sup> [https://github.com/avincesmedile/YREC-Wrappers/blob/main/modelgrid\\_tools/alternate\\_tools/change\\_nml.py](https://github.com/avincesmedile/YREC-Wrappers/blob/main/modelgrid_tools/alternate_tools/change_nml.py)

#### A.2.1. YREC Output Reader

YREC returns a set of output files, such as the `.track` files, which detail the evolution of the star from the designated starting point. The ability to read batches of such files can be useful. We provide functionality to locate and read large batches of files into Python programs for manipulation with `pandas` or `astropy`. We note that some minor modifications of the source code, discussed in the package, are required to use this option.

The wrappers themselves—`load_yrec_tracks()`<sup>20</sup> and `tracker()`<sup>21</sup>—are provided to users to read in `.track` files. The `tracker()` function can be used together with `load_yrec_tracks()` or on its own. Users provide a filepath to a `.track` file and it is read into Python as a `Pandas DataFrame`. The `load_yrec_tracks()` function takes a filepath to a folder containing all the outputted files. The code either uses the immediate directory or searches all subdirectories for `.track` files (to be used if the `nml` were modified using `change_nml()` function). Optional functionality is given to manipulate the stored data to create `EEPs` and `Isochrones` and return them alongside the main dataset.

#### A.3. Model Grid Generators

We provide wrappers that can be used to load the input physics and namelists and execute YREC to generate various grids of models. We provide the means to recreate our sample model grid (`make_modelgrid.py`)<sup>22</sup>, as well as generate a grid of models with the same input physics but a range of masses and metallicities via Python (`batchrunner.py`)<sup>23</sup> and Slurm (`run_yrec_grid.slurm`)<sup>24</sup> scripts to run a grid. In ad-

<sup>20</sup> [https://github.com/avincesmedile/YREC-Wrappers/blob/main/modelgrid\\_tools/main\\_tools/load\\_yrec\\_tracks.py](https://github.com/avincesmedile/YREC-Wrappers/blob/main/modelgrid_tools/main_tools/load_yrec_tracks.py)

<sup>21</sup> [https://github.com/avincesmedile/YREC-Wrappers/blob/main/modelgrid\\_tools/main\\_tools/Tracker.py](https://github.com/avincesmedile/YREC-Wrappers/blob/main/modelgrid_tools/main_tools/Tracker.py)

<sup>22</sup> [https://github.com/avincesmedile/YREC-Wrappers/blob/main/modelgrid\\_tools/main\\_tools/make\\_modelgrid.py](https://github.com/avincesmedile/YREC-Wrappers/blob/main/modelgrid_tools/main_tools/make_modelgrid.py)

<sup>23</sup> [https://github.com/avincesmedile/YREC-Wrappers/blob/main/modelgrid\\_tools/main\\_tools/batchrunner.py](https://github.com/avincesmedile/YREC-Wrappers/blob/main/modelgrid_tools/main_tools/batchrunner.py)

<sup>24</sup> [https://github.com/avincesmedile/YREC-Wrappers/tree/main/modelgrid\\_tools/slurm.tools](https://github.com/avincesmedile/YREC-Wrappers/tree/main/modelgrid_tools/slurm.tools)

2190 dition to this, we also provide the `yrec_parallel.py`<sup>25</sup> 2235  
 2191 script for running large batches of models on differ- 2236  
 2192 ent CPUs of the users' native machine. This tool sup- 2237  
 2193 ports systematic variation of input values to explore a 2238  
 2194 given physics parameter space across many independent 2239  
 2195 YREC runs. For all batch scripts, users provide the path 2240  
 2196 information for their YREC installation and namelists, 2241  
 2197 then specify the masses and Fe/H they wish the grid to 2242  
 2198 cover. The codes run the YREC executable with the cor- 2243  
 2199 responding namelists. Forthcoming additions include an 2244  
 2200 option to generate ranges of birth rotation rates, which 2245  
 2201 requires custom nml2 files as well as custom nml1 files. 2246

#### 2202 A.4. Solar Model Calibration

2203 Our solar model calibrations illustrate the power 2204  
 2205 of scripting YREC execution. We provide the 2206  
 2207 `solar_rot_calibrated.py` script<sup>26</sup> to this end. The 2208  
 2209 code performs a template solar model run, including ro- 2210  
 2211 tation, with user-specified initial X, Z, mixing length  $\alpha$ , 2212  
 2213 birth rotation rate, angular momentum loss scale factor 2214  
 2215  $f_k$ , and mixing efficiency parameter  $f_c$ . The user also 2216  
 2217 specifies the solar M, Z/X, L, R, age, surface rotation 2218  
 2219 period, and the target solar lithium depletion. In the 2220  
 2221 classical or diffusion model case, the script evaluates L 2222  
 2223 and R, and iterates X, Z, and  $\alpha$  until they match the sol- 2224  
 2225 ar values within a user-specified tolerance. This is done 2226  
 2227 with partial derivatives of L and R with respect to X and 2228  
 2229  $\alpha$  taken from prior solar model runs; Z is adjusted to the 2230  
 2231 user-specified Z/X. Users can also specify their own par- 2232  
 2233 tial derivatives if desired. In the case where diffusion is 2233  
 2234 included, the script adds a check on the surface Z/X to 2234  
 2235 ensure that it matches the user-specified current value. 2235  
 2236 In a classical model, the birth value is the same as the 2236  
 2237 current value, so no such check is performed. Finally, 2237  
 2238 the model with rotation adds the requirement that the 2238  
 2239 model surface rotation rate and logarithmic lithium de- 2239  
 2240 pletion match the solar values. This is done by adjust- 2240  
 2241 ing the zero point of the angular momentum loss rate 2241  
 2242  $f_k$  and the efficiency of mixing with respect to angular 2242  
 2243 momentum transport  $f_c$ , respectively. 2243

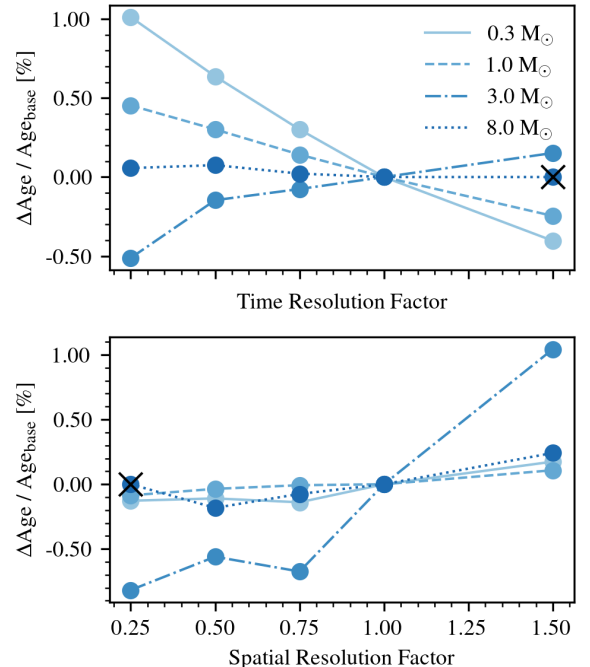
#### 2229 A.5. Tolerance Calibration

2230 These scripts perturb the spatial and temporal resolu- 2231  
 2232 tion of a specified run and infer the change in a figure of 2232  
 2233 merit. This can be used to define the numerical controls 2233  
 2234 required to meet a goal. In Figure 22 we show an exam- 2234  
 2235 ple for the models used to construct the base grid; in this 2235

<sup>25</sup> [https://github.com/avincesmedile/YREC-Wrappers/blob/main/modelgrid\\_tools/alternate\\_tools/yrec\\_parallel.py](https://github.com/avincesmedile/YREC-Wrappers/blob/main/modelgrid_tools/alternate_tools/yrec_parallel.py)

<sup>26</sup> [https://github.com/avincesmedile/YREC-Wrappers/blob/main/modelgrid\\_tools/main\\_tools/solar\\_rot\\_calibrated.py](https://github.com/avincesmedile/YREC-Wrappers/blob/main/modelgrid_tools/main_tools/solar_rot_calibrated.py)

2235 case, our figure of merit was the age at TAMS. We rec- 2236  
 2237 ommend using the `update_nml()` or `change_nml()` func- 2238  
 2239 tions to make a model run and change the spatial and 2239  
 2240 temporal tolerances as we do in Figure 22, and choose 2240  
 2241 a metric of merit (such as Age in our case) to compare 2241  
 2242 the end-of-run values to. 2242



**Figure 22.** Spatial and temporal resolution tests for solar metallicity models at 0.3, 1.0, 3.0, and 8.0  $M_{\odot}$ . The tolerances from the grid namelists were multiplied by the indicated factors, and the resulting percent change in the TAMS age is show relative to the base-resolution model. X's denote failure to converge.

## 2244 B. TROUBLESHOOTING YREC

2245 The YREC code will fail to execute in some cases, 2246  
 2247 and it can be helpful to understand the origin of some 2247  
 2248 of the failure modes. The `.short` output file contains 2248  
 2249 very useful diagnostic data in case of a failure, and it 2249  
 2250 should be consulted. Here are some common problems 2250  
 2251 and solutions: 2251

### 2248 B.1. Failure to converge

2249 The code will apply corrections to the structure vari- 2250  
 2251 ables iteratively. If it has not converged within the tol- 2251  
 2252 erances in a user-specified number of attempts, it will 2252  
 2253 stop. Sometimes this can be solved by simply increas- 2253  
 2254 ing the maximum iterations; this may be fruitful if the 2254  
 2255 corrections are decreasing slowly in amplitude. More of- 2255  
 2256 ten, they reach a flat amplitude and change sign. The 2256

most common cause is that the spatial or temporal tolerances are too loose, such that the solution cannot reach the desired precision. In some cases, changing the size of the envelope triangle can also help, as this influences the accuracy of the surface boundary condition interpolator. Users may also relax the convergence criteria; be cautioned, however, that tolerances looser than the  $10^{-4}$  level can produce erratic behavior in global properties.

Sometimes the code will cascade down to an extremely small timestep, which always results in a convergence failure. In this case, the solution is counter-intuitive: the timestep criteria need to be relaxed. In the limit of very small timesteps, the corrections become very sensitive to numerical noise.

The third and fourth levels of iteration can produce convergence failures in evolved red giant branch models; we recommend that users set these to 0 unless they are explicitly needed to study rotational mixing.

### B.2. *Hard failure modes.*

Other failures are tied to the assumptions in the code, and they are not numerical in nature. The current code fails if both overshoot and semi-convection are used at the same time. Above  $\sim 16$  solar masses on the main sequence, gray atmospheres models will have a net negative gas pressure due to the onset of radiation pressure driven winds, which YREC does not currently account for. The code will also fail if rotation is so rapid that material at the equator is unbound; users should check that the internal rotation profile does not fail this criterion. Frequently this condition is triggered by a poor choice of starting conditions (see below). Finally, the code cannot evolve through a degenerate helium flash. This failure mode will manifest as runaway corrections, and it cannot be solved without changes to the code itself.

### B.3. *Challenging domains*

Stellar physics is the most uncertain at very high and very low masses. We are actively working on the code, and anticipate progress on the phase space where it can be used. Below we document the current domains where YREC is either difficult to run or not well suited given its current assumptions.

For high mass stars, a combination of missing physics and numerical challenges makes it difficult to run models above 9 solar masses through core He exhaustion.

Models with rotation can be run across the full mass range. However, rotationally induced mixing is too efficient for stars above the Kraft break, and we do not recommend running models with mixing in this domain.

Models with microscopic diffusion encounter numerical difficulties when the hydrogen or helium mass frac-

tions approach zero in the envelope. We therefore also recommend that the code not be used with diffusion enabled above  $\sim 1.4$  solar masses at solar metallicity.

Brown dwarfs and stars close to the hydrogen-burning limit are numerically challenging, both due to the uncertain EoS and the importance of the surface boundary conditions for the solution. We do not recommend using the Yale EoS below  $0.5M_{\odot}$ , or using a gray atmosphere below  $0.3M_{\odot}$ . The [F. Allard & P. H. Hauschildt \(1995\)](#) grid is the currently recommended choice for low mass stars.

We also note that models close to  $0.1M_{\odot}$  can encounter challenges because their physical conditions skirt close to the edge of published equation of state tables, making their structure sensitive to the ramping between different EoS sources.

Core helium burning stars may not achieve  $Y_{\text{cen}} = 10^{-4}$ , reaching temperatures that may be outside the bounds of interior opacity tables before reaching low central helium abundance. The equilibrium solution for the seed ZAHB models used for the low-mass two-step rescaling procedure is also sensitive to the choice of neutrino cooling physics, and users are cautioned that their model may not run if attempting to use [N. Itoh et al. \(1996\)](#) rates, which are otherwise the standard for YREC. All low-mass core helium burning model namelists in the test suite set LNULOS1 to false, which results in defaulting to rates from [J. N. Bahcall \(1989\)](#).

### B.4. *Choices that can lead to sorrow.*

Our digital constructs lack imagination and foresight, merely doing what they are asked to do. It is therefore in the nature of software to fail. Some of these failures reflect poor design. There are times, however, where the fault lies not in the stars, but in ourselves ([Shakespeare 1599](#)). Here we list, from personal experience, lessons that users of the code should take to heart.

There may be a time when computers can read our minds, doing the things that we wish to be done. We do not yet live in such a time. If you do not tell YREC where to read from, or where to write to, it will fail. Check your pointers.

The code features a lovely variety of options, but human imagination is far greater in scope. If you include variables that the code does not recognize in the namelists, it will quit in disgust. Ensure that your comments are clearly labelled as such, and that the version of the code that you are using supports the choices that you are making.

It is tempting to use a single template to clone a series of runs, changing the output pointers to produce large grids of models. It is important to check that you are

2358 also changing the inputs, such that the files that you  
2359 produce are what you think they are. In a better world,  
2360 you would always get what you desire. In this one, you  
2361 get what you deserve.

2362 The code does, in fact, provide diagnostic outputs.  
2363 When it fails, or misbehaves, reading the messages on  
2364 the screen, or those stored in the short file, can lead to  
2365 enlightenment. Or not, of course.

2366 If you assign the wrong input to the wrong name, the  
2367 code will become confused at this offense to the natural  
2368 order of things and quit. However, if you assign a file  
2369 with the right format, but the wrong physics, to the  
2370 right place—say, for example, an atmosphere table with  
2371 a metal content different from that of the star—it will  
2372 assume that you have wisdom, and it will allow you to  
2373 proceed without comment. Ask yourself whether this  
2374 trust is warranted.

2375 Conservation laws are the foundation of modern  
2376 physics, and YREC respects these iron rules. If you  
2377 start with a very large star, and give it a modest spin, it  
2378 will quickly become unruly as it shrinks and spins up. At  
2379 some point it may become unbound at the equator—a

2380 situation that YREC finds intolerable, as it cannot con-  
2381 ceive of imaginary numbers. Check your starting point  
2382 for models with rotation. It is easy to give stars far more  
2383 angular momentum than they can find a use for.

2384 While we are on the subject of imaginary numbers,  
2385 recall that the code uses logarithmic variables. Zero  
2386 is a special number. Before you set something in the  
2387 code to zero, some introspection is required. Do you  
2388 really need that number to be zero, or will a very small  
2389 number suffice instead? Do you feel lucky? Be prepared  
2390 to experiment and learn. In some cases, the code will  
2391 anticipate folly and quit. In other, more devious cases,  
2392 it will not, and it may break in unusual ways.

2393 Finally, a successful run of the code is the beginning,  
2394 not the end, of the journey. You should plot your re-  
2395 sults and see if they appear sensible. You should change  
2396 your numerics and see whether they change your answer  
2397 to any interesting degree. If they do, ponder your life  
2398 choices, and adjust. Ensure that the tables that you use  
2399 are the ones you actually want to use. Ensure that the  
2400 physics that you think that you are using are the same  
2401 as the physics that you are, in fact, using.

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